- I Jaen Conference on Approximation Theory
- Úbeda, Jaén, Spain, July 4th-9th, 2010
- Approximation Theory, Computer Aided Geometric Design, Numerical Methods and Applications

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Introduction

The Conference is a new activity of the Jaen Approximation Project. Jaen Approximation Project has organized ten editions of the Ubeda Meeting on Approximation and issues the Jaen Journal on Approximation. The objective of this conference is to provide a useful and nice forum for researchers in the subjects to meet and discuss. In this sense, the conference program has been designed to keep joined the group during five days with a program full of scientific and social activities. The Conference will be devoted to some significant aspects on Approximation Theory, Computer Aided Geometric Design, Numerical Methods and the Applications of these fields in other areas.

The Conference features six invited speakers who will give one-hour plenary lectures. Researchers were invited to contribute with a talk or a poster. We scheduled almost 90 short talks and a poster session. We also provided the possibility to organizing mini-symposia on a subject of current interest. The following proposals have been accepted: "Asymptotic Methods in Analysis" organized by Diego Dominici, and "Non standard Orthogonal Polynomials" organized by Francisco Marcellán, Teresa E. Pérez, and Miguel A. Piñar.

The conference will include a special session dedicated to Prof. Mariano Gasca on the occasion of his retirement. Prof. Mariano Gasca, a relevant Spanish mathematician, is very related to Jaén, for instance, he was the first Director of the Jaén branch of the University of Granada, he is a member of the editorial board of Jaen Journal on Approximation, member of the Scientific Committee of Ubeda Meeting on Approximation and he has many academic descendants in Jaén. Professor Mariano Gasca retired in 2009 and now he is 'Emeritus Professor' of the University of Zaragoza, Spain. He is a Spanish pioneer in the research of numerical methods and the quality of his results is backed by almost one hundred papers in the most prestigious journals of the field.

The Conference will take place at the Cultural Center Hospital de Santiago in Úbeda, a World Heritage Site. The opening ceremony and the special session will take place on July 5th at 9:00 a.m.

Finally, but also important, the Conference provides to participants the possibility to visit World Heritage Sites and taste a wide culinary variety. We will do all the best for accompanying people to enjoy the Conference.

We are grateful to all those who have made this project a reality; the University of Jaén (Vicerrectorado de Investigación, Facultad de Ciencias Experimentales, Facultad de Ciencias Sociales y Jurídicas, Departamento de Matemáticas), Ministerio de Ciencia e Innovación (MTM2009-08236-E), Junta de Andalucía, Diputación Provincial de Jaén, Obra Socio Cultural Caja de Jaén, Ayuntamiento de Úbeda, Agua Sierra de Cazorla, Fundación Cruzcampo, Caja Rural Jaén, Ayuntamiento de Baeza, Castillo de Canena, Fundación del Olivar, Iloveaceite.com, UNIA. Here we emphasize our commitment to keep on working to improve our university and our province.

The Organizing Committee

	July, 5th-Monday		
	ROOM A	**ROOM B**	**ROOM C**
9:00-	**Opening Ceremony and Special Session Dedicated to Mariano Gasca**		
	Victoriano Ramírez (p. 6)		
	Guenter Mühlbach (p. 3)		
	Mariano Gasca (p. 3)		
11:00-11:20	**Coffee Break**		
11:20-11:45	Adhemar Bultheel (p. 38)	Charles Chui (p. 49)	Ana Matos (p. 119)
11:45-12:10	Luis Enrique Garza Gaona (p. 87)	Jozsef Szabados (p. 167)	Bernhard Beckermann (p. 26)
12:10-12:35	Victor Luis Sousa (p. 162)	Andras Kroo (p. 109)	David Benko (p. 29)
12:35-13:00	Wolfgang Erb (p. 77)	Igor Shevchuk (p. 157)	Mehrdad Simkani (p. 161)
13:00-13:10	**Break**		
13:10-13:35	Kathy Driver (p. 76)	Borislav Draganov (p. 63)	H. Michael Möller (p. 125)
13:35-14:00	M. Alicia Cachafeiro (p. 43)	Gert Tamberg (p. 171)	Annie Cuyt (p. 54)
14:00-14:25	Samuel Gómez Moreno(p. 126)	Andi Kivinukk (p. 104)	Michael Floater (p. 86)
14:25-14:45	Maria das Neves Rebocho (p. 143)	José María Almira (p. 22)	Marie-Laurence Mazure (p. 121)

	July, 6th-Tuesday		
	ROOM A	**ROOM B**	**ROOM C**
	PLENARY LECTURES		
9:00-10:00	Doron S. Lubinsky (p. 14)		
10:00-11:00	Jesús M. Sanz Serna (p. 16)		
11:00-11:20	**Coffee Break**		
11:20-11:45	Jeffrey Geronimo (p. 89)	Giuseppe Mastroianni (p. 118)	Paul Sablonnière (p. 150)
11:45-12:10	Alberto Ibort (p. 94)	Incoronata Notarangelo (p. 136)	Jordi Marzo (p. 117)
12:10-12:35	Cleonice F. Bracciali (p. 36)	Vita Leonessa (p. 110)	Jesus Illán (p. 96)
12:35-13:00	Manuel Alfaro (p. 21)	Ryozi Sakai (p. 151)	Francesco Dell'Accio (p. 68)
13:00-13:10	**Break**		
13:10-13:35	Lidia Fernández (p. 84)	Stefano De Marchi (p. 56)	Veronika Pillwein (p. 140)
13:35-14:00	José Manuel Rodríguez (p. 145)	Ting Hu (p. 90)	Edmundo J. Huertas Cejudo (p. 92)
14:00-14:25	Bujar Fejzullahu (p. 82)	Lei Shi (p. 158)	Ferenc Tòokos (p. 174)

	July, 7th-Wednesday		
	ROOM A	**ROOM B**	**ROOM C**
	PLENARY LECTURES		
9:00-10:00	Vilmos Totik (p. 17)		
10:00-11:00	Francesco Altomare (p. 11)		
11:00-11:20	**Coffee Break**		
11:20-11:45	Andrei Martinez-Finkelshtein (p. 116)	Yuriy Volkov (p. 179)	Avram Sidi (p. 159)
11:45-12:10	Renato Spigler (p. 164)	Manuel Calvo (p. 45)	Juergen Müller (p. 131)
12:10-12:35	Jose L. López (p. 112)	Đinh Dũng (p. 72)	Thierry Meyrath (p. 122)
12:35-13:00	Alfredo Deaño (p. 61)	Peter Binev (p. 30)	Konstantin Fedorovskiy (p. 80)
13:00-13:10	**Break**		
13:10-13:35	**POSTER SESSION**		
13:35-14:00	Herbert Stahl (p. 165)	Miguel Jiménez (p. 101)	Peter Beise (p. 28)
14:00-14:25	Thorsten Neuschel (p. 135)	Ioan Gavrea (p. 88)	André Boivin (p. 32)
14:25-14:40	Steven Delvaux (p. 70)	Dimitru Mircea Ivan (p. 98)	Pablo Manuel Román (p. 148)

	July, 8th-Thursday		
	ROOM A	**ROOM B**	**ROOM C**
	PLENARY LECTURES		
9:00-10:00	Nira Dyn (p. 13)		
10:00-11:00	Yuan Xu (p. 18)		
11:00-11:30	**Coffee Break**		
11:30-11:55	Kerstin Jordaan (p. 102)	Graziana Musceo (p. 132)	Sandra Saliani (p. 154)
11:55-12:20	Francisco Perdomo Pío (p. 138)	Sabina Milella (p. 123)	Elisabetta Santi (p. 155)
12:20-12:45	Stamatis Koumandos (p. 107)	Sabrina Diomede (p. 74)	Mikhail Tyaglov (p. 175)
12:45-13:10	Luis Cotrim (p. 52)	Mirella Cappelletti Montano (p. 47)	Sergiy Borodachov (p. 34)
13:10-13:20	**Break**		
13:20-13:45	Karl Deckers (p. 64)	Anton Makarov (p. 114)	Eduardo Corbacho Cortés (p. 50)
13:45-14:10	Isabel I. Cação (p. 41)	Vesselin Vatchev (p. 177)	Geraldo Soares de Souza (p. 60)
14:10-14:35	Carla Cruz (p. 53)	Michael Prophet (p. 142)	

Mini-symposia

Miny-symposium ’Asymptotic Methods in Analysis’

7th-Wednesday, 11:20–14:40, Room A.

Organized by Diego Dominici

Miny-symposium ’Non standard Orthogonal Polynomials’

6th-Tuesday, 11:20–14:25, Room A.

Organized by Francisco Marcellán, Teresa E. Pérez, and Miguel A. Piñar

Poster Session

7th-Wednesday, 13:10–13:35, Patio, Hospital de Santiago.

Index

Dedicated to Mariano Gasca

Plenary Talks

Short Talks/Posters

Dedicated to Mariano Gasca

Aitken, Neville and beyond. A tribute to Mariano Gasca.

Günter W. Mühlbach

Abstract

Beginning with the classical recursions for polynomial interpolation by *Aitken* and *Neville* we consider their algebraic fundamentals and some of their generalizations. The discussed topics primarily originate from scientific contributions by *Mariano Gasca*.

Keywords: Interpolation, elimination strategies, total positivity, determinantal identities.

AMS Classification: 41A05, 65D05, 65F40.

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Mariano Gasca, University of Granada 1977-1982

Victoriano Ramírez González

Professor Mariano Gasca arrived to the University of Granada in September 1977 and he was the Director of the Department of Functional Equations during the five years he stayed in this university.

From the first moment, Mariano won everybody's affection in our department as well as from the people working in the different departments of mathematics and many other faculty and staff from the rest of the University.

The affection from his disciples goes far beyond the academic. On a personal level, Mariano is a person committed to those around him, affable, he has an exquisite manner and fine humor. When he tells an anecdote (and he has many), everyone is silent, because even in that he is good.

His intense dedication to the University, both in teaching and research, highlighted above usual.

With regard to teaching, he was the only professor who have published all the material used in class. Before the arrival of Professor Gasca to Granada, Numerical Analysis had been a course which had been through many different departments of Mathematics. Teachers who had taught it were of Statistics on some occasions, and Functional Analysis in others. The N.A. had been an unpleasant subject for students and teachers who taught it in the 70s. With M. Gasca that changed radically just in a few years after his arrival, the best students in each promotion wished to enter in the Department of Functional Equations to work with him in N.A.

With regard to researching, during the five courses that Professor Gasca was in Granada, five doctoral theses directed by him were read. Two of them, Maeztu's and Cordon's had been initiated in El País Vasco and were completed and read in 1979 in Granada. In 1980 the thesis of V.Ramírez was read and he was followed by A.Lopez in 1981 and finally E. Lebron in 1982.

Thus, in those five years, Professor Gasca significantly improved the quality of teaching of NA and formed a research group in this field. It should be noted that he sowed great interest in NA to these doctors, instilling the importance of visits to other research centers, the assistance to Congress and sending articles to magazines, as well as directing doctoral theses in NA.

Fruit of all this teaching, meant that the small group of doctors in NA that was at the University of Granada when Professor Gasca returned to his hometown, Zaragoza, cared to continue its work. On one hand this small group established contacts with other national and international researchers in Numerical Analysis (Paul Sablonniere of University of Lille in France, the Francisco Marcellán of the University Carlos III of Madrid, Maria Cruz López de Silanes from the University of Zaragoza, Miguel Marano of the University of Rio Cuarto in Argentina, etc..) all of them, along with the Granada group have contributed to the creation of the existing research groups in NA at the Universities of Granada and Jaen. There are more than 20 doctors in NA between these two universities, where Prof. Gasca has presided the tribunal of qualification in several occasions. Research that is supported by a large number of publications in numerical, representing one of three or four most important groups in Spain. All this has been due to the residence of Professor Gasca at the University of Granada and his collaboration after his departure to Zaragoza.

As a representative sample of the scientific heritage is interesting to note that the promoter of this Congress and the magazine just born, Professor F.J. Muñoz, is precisely an academic grandchild of Professor Gasca. Surely if M. Gasca had not come to Granada this Congress would not exist and probably nobody would be researching NA Granada or Jaén.

In terms of management, M. Gasca was a person called to participate at all levels. In fact he was the first Director of the university college in Jaén (when there was no highway and you had to go through the mountain pass of El Carretero, el Zegrí, Onítar, etc.) He was the director of the Section of Mathematics at the University of Granada. He rejected the offer to become vice-rector for Research. It's been his wisdom, humility, honesty and dedication what has brought to all those who have known him to offer him the highest positions in the academic responsibility. An example to emulate.

The universities of Granada and Jaén have benefited greatly from the stay of M. Gasca here.

Thanks Mariano, all we have much to thank you, and I more than anyone.

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Plenary Talks

• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Asymptotic formulae and positive semigroups of operators

Francesco Altomare

Abstract

The study of asymptotic formulae for positive linear operators is a topic of current interest in approximation theory. Such formulae play an important role in the analysis of the saturation properties of positive approximation processes and in the approximation of strongly continuous positive semigroups by means of iterates of operators.

A typical situation occurs when dealing with a sequence $(L_n)_{n\geq 1}$ of positive linear operators defined on a Banach lattice E of continuous functions, which satisfies an asymptotic formula of the form

$$\lim_{n\to\infty} n(L_n(f)-f) =: Z(f) \text{ strongly on } E$$

for every f belonging to a suitable subspace D_0 of E. Under additional hypotheses it can be shown that there exists a dense linear subspace $D(A)$ of E such that $D_0 \subset D(A)$, A is an extension of Z and the operator $(A, D(A)$ generates a strongly continuous C_0-semigroup $(T(t))_{t\geq 0}$ of operators on E such that

$$\lim_{n\to\infty} L_n^{k(n)}(f) = T(t)f \qquad \textit{for every } f \in E \textit{ and } t \geq 0,$$

where $(k(n))_{n\geq 1}$ is an arbitrary sequence of positive integers such that $k(n)/n \to t$. In particular this means that, given $u_0 \in E$, the Cauchy problem

$$\begin{cases} u'(t) = Au(t) & t \geq 0, \\ u(0) = u_0 \end{cases}$$

has a unique solution if and only if $u_0 \in D(A)$ and, moreover, this solution is given by

$$u(t) = T(t)u_0 = \lim_{n\to\infty} L_n^{k(n)}(u_0) \qquad (t \geq 0).$$

In spite of the prominent interest of $(A, D(A)$, in concrete situations it is rather complicate to determine it. Recently ([1], [2]) a general principle was discovered which stresses

a new relationship between $(L_n)_{n\geq 1}$ and $(A, D(A)$. More precisely, under suitable assumptions, for every $u_0 \in E$, the following properties are equivalent:

(i) $u_0 \in D(A)$;

(ii) *there exists* $v_0 \in E$ *such that* $\lim_{n\to\infty} n(L_n(u_0) - u_0) = v_0$ *pointwise.*

In such a case, moreover, $A(u_0) = v_0$.

This result can be useful to determine $D(A)$ by means of asymptotic formulae. Conversely, if $D(A)$ is known, it allows to infer some smoothness properties (which are usually incorporated in the domain $D(A)$) for those functions satisfying the pointwise asymptotic formula (ii).

The talk will be devoted to this circle of ideas. Several applications will be also discussed which concern approximation processes on bounded as well as unbounded intervals such as Bernstein operators, Kantorovich operators, exponential operators (including Szász-Mirakjan, Baskakov and Post-Widder operators), generalized Gauss-Weierstrass operators and a sequence of operators related to the Black-Scholes equation, which is of interest in Mathematical Finance.

For these sequences of positive operators some local saturation result will be also shown.

Keywords: Positive approximation process, asymptotic formula, C_0-semigroup of operators, local saturation.

AMS Classification: 41A36, 41A40, 41A80, 47D07.

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Approximation of set-valued functions

Nira Dyn

Abstract

This talk reviews results on the approximation of univariate set-valued functions by samples-based linear approximation operators. The approach is to adapt samples-based linear approximation operators for real-valued functions to set-valued functions, by replacing operations between numbers by operations between sets. For set-valued functions with compact convex images, it has been known that adaptations of positive operators to sets, based on Minkowski convex combinations of sets yield approximating operators in the Hausdorff metric. Yet for set-valued functions with general compact images, it can be shown that this adaptation when applied to certain positive operators, results in operators which fail to approximate these functions.

We present approximation results in the Hausdorff metric for set-valued functions with general compact images, obtained with samples-based approximation operators, adapted to sets with the metric average or with its extension to metric linear combinations of sets. We apply the general results to two types of positive operators, the Bernstein polynomial operators and the Schoenberg spline operators. For adaptations based on metric linear combinations, the general results are appllicable to approximation operators which are not necessarily positive. In this case we apply the results also to polynomial interpolation operators.

This talk reports on joint works with E. Farkhi and A. Mokhov.

Keywords: Compact sets, set-valued functions, linear approximation operators, positive linear approximation operators, error estimates, Minkowski sum of sets, metric average, metric linear combinations.

AMS Classification: 26E25, 41A35, 41A36, 47H04, 54C65.

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New (??) identities for orthogonal polynomials on the real line*

Doron S. Lubinsky

Abstract

Let μ be a positive measure on the real line, with orthonormal polynomials $\{p_n\}$ and leading coefficients $\{\gamma_n\}$. There are a number of apparently new identities for these, that have roots in the theory of de Branges spaces. Amongst these are:

(I) A Geronimus type formula, valid for all polynomials P of degree $\leq 2n-2$: let $z \in \mathbb{C}\backslash\mathbb{R}$. Then for all polynomials P of degree $\leq 2n-2$,

$$\frac{1}{\pi}\left|\mathrm{Im}z\right| \int_{-\infty}^{\infty} \frac{P(t)}{\left|zp_n(t)-p_{n-1}(t)\right|^2} dt = \frac{\gamma_{n-1}}{\gamma_n} \int P d\mu.$$

For one particular value of z, this gives a formula due to Barry Simon, which he called a Carmona type formula.

(II) A more general formula: let $h \in L_1(\mathbb{R})$. Then for all polynomials P of degree $\leq 2n-2$,

$$\int_{-\infty}^{\infty} \frac{P(t)}{p_n(t)^2} h\left(\frac{p_{n-1}(t)}{p_n(t)}\right) dt = \frac{\gamma_{n-1}}{\gamma_n}\left(\int_{-\infty}^{\infty} h\right) \int P d\mu.$$

(III) An explicit formula, of Bernstein-Szegő type, for the orthogonal polynomial of degree n, when

$$\mu'(x) = \frac{dx}{S(x)}, \quad x \in (-\infty, \infty)$$

and S is a positive polynomial of degree $2n+2$.

We shall discuss these, their relation to Gauss quadrature, and some consequences.

Keywords: Orthogonal polynomials, identities.
AMS Classification: 42C05.

*Research supported by NSF grant DMS0700427 and US-Israel BSF grant 2008399.

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Optimal tuning of hybrid Monte Carlo

Jesús María Sanz Serna

Abstract

I will begin by presenting a simple introduction to Markov Chain Monte Carlo methods, a family of algorithms that feature amongst the most widely used in all mathematics. I will then move to recent research (joint with A. Beskos, N. S. Pillai, G. O. Roberrts and A. M. Stuart) investigating the properties of one such algorithm, the Hybrid Monte Carlo method, when the dimension d of the state space is large. In the simplified scenario of independent identically distributed components we prove that to obtain $\mathcal{O}(1)$ acceptance probability as d tends to infinity, the time-step h of the Verlet algorithm used to simulate the Hamiltonian dynamics must be scaled as $d^(1/4)$. We also identify the universal optimal value of the acceptance probability.

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Polynomials with minimal norms

Vilmos Totik

Abstract Consider a compact set E on the complex plane and the associated monic polynomials of degree n with minimal supremum norms on E (Chebyshev polynomials). If their minimal norm is t_n, then it is classical that $\{t_n^{1/n}\}$ has a limit which equals the transfinite diameter $\tau(E)$ of E (and also equals the logarithmic capacity of E). It is always the case that $t_n \geq \tau(E)^n$. In this talk we are interested in how close one can get with t_n to the lower bound $\tau(E)^n$, and some recent results concerning disconnected sets, sets consisting of finitely many intervals or finitely many Jordan curves will be mentioned. Another topic that we shall discuss is that if $P_n(z) = z^n + \cdots$ are monic polynomials which have k_n zeros on E or on some part of it, then how close its norm can be to the theoretical lower bound $\tau(E)^n$ (say, for E the unit circle or some families of Jordan curves). The unit circle case of this latter problem is coming from Turán's power-sum method in number theory. One of the results is the following: if P_n has $n|J|/2\pi + k_n$ zeros on a subarc J of the unit circle E, then $\|P_n\|_E \geq c\exp(ck_n^2/n)$, and this estimate is exact. Some of the results that will be mentioned are from a joint work with Péter P. Varjú.

Keywords: Chebyshev polynomials, disconnedted sets, norms of minimal polynomials
AMS Classification: 41A10, 31A15.

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Moduli of smoothness and approximation on the unit sphere and the unit ball

Feng Dai and Yuan Xu

Abstract

A new modulus of smoothness based on the Euler angles is introduced on the unit sphere and is shown to satisfy all the usual characteristic properties of moduli of smoothness, including direct and inverse theorem for the best approximation by polynomials and its equivalence to a K-functional, defined via partial derivatives in Euler angles. The set of results on the moduli on the sphere serves as a basis for defining new moduli of smoothness and their corresponding K-functionals on the unit ball, which are used to characterize the best approximation by polynomials on the ball.

Keywords: Modulus of smoothness, K-functional, best approximation, sphere, Euler angles, ball.

AMS Classification: 42B15, 41A17, 41A63.

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Short Talks/Posters

• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

On a conjecture concerning Laguerre-Sobolev type orthogonal polynomials*

M. Alfaro, J.J. Moreno-Balcázar, A. Peña and M.L. Rezola

Abstract

We solve the conjecture established in [1] concerning Mehler-Heine formula for Laguerre-Sobolev type orthogonal polynomials. As a consequence, the convergence of the zeros of these polynomials towards the origin is studied.

Keywords: Laguerre-Sobolev type polynomials, Mehler-Heine formulas, zeros, asymptotics, Bessel functions.

AMS Classification: 42C05, 33C45.

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*Research supported by MICINN of Spain (grants MTM2009-12740-C03-03 and MTM2008-06689-C02-01), Diputación General de Aragón (project E-64) and Junta de Andalucía (FQM229 and excellence project P06-FQM-1735).

• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Shapiro's Theorem for subspaces

J. M. Almira and T. Oikhberg

Abstract

Let $(X, \|\cdot\|)$ be a quasi-Banach space, and let $A_0 \subset A_1 \subset \cdots \subset A_n \subset \cdots \subset X$ be an infinite chain of subsets of X, where all inclusions are strict. We say that $(X, \{A_n\})$ is an *approximation scheme* (or that (A_n) is an approximation scheme in X) if:

(i) There exists a map $K : \mathbb{N} \to \mathbb{N}$ such that $K(n) \geq n$ and $A_n + A_n \subseteq A_{K(n)}$ for all $n \in \mathbb{N}$.

(ii) $\lambda A_n \subset A_n$ for all $n \in \mathbb{N}$ and all scalars λ.

(iii) $\bigcup_{n\in\mathbb{N}} A_n$ is a dense subset of X.

A particular example is a *linear approximation scheme*, arising when the sets A_n are linear subspaces of X. In this setting, we can take $K(n) = n$. An approximation scheme is called *non-trivial* if $X \neq \overline{\cup_n A_n}$.

Approximation schemes (introduced by Y. Brudnyi and N.Kruglyak under the name of "approximation families" in 1978 [7] and popularized by A. Pietsch in [10]) are a natural object to study in Approximation Theory. Indeed, the approximation scheme concept is the abstract tool that models all approximation processes, and can be considered as a central concept for the theory.

In our previous paper [2] approximation schemes $(X, \{A_n\})$ with some lethargy property were investigated. These results have their origin in a classical paper by S.N. Bernstein [6], and guarantee the existence of elements $x \in X$ with a sequence of best approximation errors $\{E(x, A_n)\}_{n=0}^{\infty}$ that decays to zero as slow as we want. In [2] we gave several characterizations of the approximation schemes with the property that for every non-increasing sequence $\{\varepsilon_n\} \searrow 0$ there exists an element $x \in X$ such that $E(x, A_n) = \inf_{a\in A_n} \|x - a\| \neq \mathbf{O}(\varepsilon_n)$ (in this case we say that $(X, \{A_n\})$ *satisfies Shapiro's Theorem*). In particular, the following result is contained in [2, Theorem 2.2, Corollary 3.6]):

Theorem 1. *Let $(X, \{A_n\})$ be an approximation scheme. Then the following claims are equivalent:*

(a) *The approximation scheme $(X, (A_n))$ satisfies Shapiro's Theorem.*

(*b*) *There exists a constant* $c > 0$ *and an infinite set* $\mathbb{N}_0 \subseteq \mathbb{N}$ *such that for all* $n \in \mathbb{N}_0$, *there exists some* $x_n \in X \setminus \overline{A_n}$ *which satisfies* $E(x_n, A_n) \leq cE(x_n, A_{K(n)})$.

(*c*) *There is no decreasing sequence* $\{\varepsilon_n\} \searrow 0$ *such that* $E(x, A_n) \leq \varepsilon_n \|x\|$ *for all* $x \in X$ *and* $n \in \mathbb{N}$.

(*d*) $E(S(X), A_n) = \sup_{x \in S(X)} E(x, A_n) = 1$, $n = 0, 1, 2, \cdots$.

(*e*) *There exists* $c > 0$ *such that* $E(S(X), A_n) \geq c$, $n = 0, 1, 2, \cdots$.

Moreover, if X *is a Banach space, then all these conditions are equivalent to:*

(*f*) *For every non-decreasing sequence* $\{\varepsilon_n\}_{n=0}^{\infty} \searrow 0$ *there exists an element* $x \in X$ *such that* $E(x, A_n) \geq \varepsilon_n$ *for all* $n \in \mathbb{N}$.

In this talk, we are interested in a "restricted version" of Shapiro's Theorem.

Definition 2. Suppose Y is a linear subspace of a quasi-Banach space X. We say that Y *satisfies Shapiro's Theorem* with respect to the approximation scheme $(X, \{A_n\})$ if, for any $\{\varepsilon_n\} \searrow 0$, there exists $y \in Y$ such that $E(y, A_n)_X \neq \mathbf{O}(\varepsilon_n)$.

By default, we view Y as a space, equipped with its own quasi-norm, and embedded continuously into X. If, in addition, Y is a closed subspace of X, Open Mapping Theorem (see [9, Corollary 1.5]) shows that the norms $\|\cdot\|_X$ and $\|\cdot\|_Y$ are equivalent on Y.

We start by giving a general description of subspaces satisfying Shapiro's Theorem. We then deal with the specific cases of boundedly compact approximation schemes and complemented subspaces. Then we give exhibit several subspaces (arising from harmonic analysis) which satisfy Shapiro's Theorem. We also show that subspaces satisfying Shapiro's Theorem are stable under small perturbations, and give some criteria for Shapiro's Theorem to be satisfied. Furthermore, we provide examples of subspaces satisfying Shapiro's Theorem and we show that the concept of smoothness associated to membership to an approximation space is of global nature. In particular, we show the following result:

Theorem 3. *Let* $0 < \alpha < \beta$ *and let* $[a, b] = [0, 1]$ *or* $[a, b] = [\alpha, \beta]$. *Let* $\{A_n\}$ *be an approximation scheme in* $C[a, b]$.

(A) If A_n *is boundedly compact for all* n *or* $\{A_n\}$ *is generalized Haar, then for all* $\{\varepsilon_n\} \to 0$ *there exists* $f \in C[a, b]$ *which is analytic in* (a, b), *such that* $E(f, A_n) \neq \mathbf{O}(\varepsilon_n)$.

(B) If A_n *is a finite dimensional space for all* n *or* $\{A_n\}$ *satisfies a generalized de La Vallée-Poussin Theorem, then for all* $\{\varepsilon_n\} \to 0$ *there exists* $f \in C[a, b]$ *which is analytic in* (a, b), *such that* $E(f, A_n) \geq \varepsilon_n$, $n = 0, 1, 2, \cdots$.

Keywords: Approximation scheme, approximation error, Bernstein's Lethargy, Shapiro's Theorem, Brudnyi's Theorem.

AMS Classification: 41A25, 41A65, 41A27.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

On orthogonal polynomials of the second category

Leffet S. and Atia M. J.

Abstract

The following conjectures were also communicated in the OPSFA meeting Leuven-Belgium 2009.

Let Q_n be the polynomial sequence orthogonal on $[-1,1]$ with respect to the weight $w_e(x) = \dfrac{|x-\frac{1}{2}|}{\sqrt{1-x}} + \dfrac{|x+\frac{1}{2}|}{\sqrt{x+1}}$ and R_n the polynomial sequence orthogonal on $[-1,1]$ with respect to the weight

$$\widetilde{w_e}(x) = |x+\frac{1}{2}|\sqrt{x+1}(1-x) + |x-\frac{1}{2}|\sqrt{1-x}(1+x)$$

then Q_n and R_n satisfy:

$$\begin{cases} Q_0(x) = 1, \quad Q_1(x) = x, \\ Q_{n+2}(x) = xQ_{n+1}(x) - \xi_{n+1}Q_n(x), \; n \geq 0, \end{cases}$$

and

$$\begin{cases} R_0(x) = 1, \quad R_1(x) = x, \\ R_{n+2}(x) = xR_{n+1}(x) - \mu_{n+1}R_n(x), \; n \geq 0. \end{cases}$$

Conjecture 1. *For $n \geq 0$, we have*

$$\xi_0 = 0, \quad \xi_{3n} + \xi_{3n+1} = \frac{1}{2} \textit{ and } \xi_{3n+2} = \frac{1}{4}.$$

Conjecture 2. *For $n \geq 0$, we have*

$$\mu_{3n+2} + \mu_{3n+3} = \frac{1}{2} \textit{ and } \mu_{3n+1} = \frac{1}{4}.$$

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Polynomial extremal problems and logarithmic potential theory in numerical linear algebra

Bernhard Beckermann

Abstract

We revisit several extremal problems for polynomials or rational functions occurring in the analysis of methods in numerical linear algebra, such as the approximation of eigenvalues by Ritz values, the convergence of iterative methods for solving systems of equations like conjugate gradients and ADI, or the approximation of matrix functions.

Subsequently we show how logarithmic potential theory can help to describe the weak asymptotics of such extremal problems.

Keywords: Rational Krylov, Ritz values, ADI method, matrix functions, orthogonal rational functions, logarithmic potential theory.

AMS Classification: 15A18, 31A05, 31A15, 65F15.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

A transform for entire functions and its application to frequently universal functions

Peter Beise

Abstract

Let $T : E \to E$ be a continuous operator, where E is some space of entire functions. A function $f \in E$ is said to be frequently universal for T if the orbit $(T^n f)$ approximates every other entire function with a certain frequency. That means, for every entire function g, every compact set $K \subset \mathbb{C}$ and every $\varepsilon > 0$, the sequence of natural numbers that satisfy $\sup_{z\in K} |T^n f(z) - g(z)| < \varepsilon$ has positive lower density.

We introduce a transform that acts on spaces of entire functions of exponential type. Therewith, we derive conditions for the "size" and the location of the conjugate indicator diagram of frequently universal functions for differential operators. This immediately yields growth properties in different directions of the complex plane for these functions.

Keywords: Frequently universal functions, differential operators, entire functions of exponential type, Borel transform.

AMS Classification: 30K99, 30D10.

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The support of the equilibrium measure

David Benko and Peter Dragnev

Abstract

Potential theory is a useful tool in approximation theory. Let E be a set of n intervals or n arcs of the unit circle. In the talk we will consider both the logarithmic and Riesz kernels and external fields defined on E. The equilibrium measure is minimizing a double integral in the presence of an external field. We show that the support of the equilibrium measure is the union of intervals/arcs, if the signed equilibrium measure satisfies certain properties. We also show that the equilibrium measure of the set E has convex density.

Keywords: potential theory, equilibrium measure, signed equilibrium measure, support.
AMS Classification: 31A15.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

On the Greedy approach to reduced basis method*

Peter G. Binev

Abstract

The reduced basis method was introduced for the accurate online evaluation of solutions to a parameter dependent family of elliptic partial differential equations. In abstract form, its goal is to determine an n dimensional space $\mathcal{H}_n$ that approximates well the elements of a compact set $\mathcal{F}$ in a Hilbert space $\mathcal{H}$. The algorithms for finding $\mathcal{H}_n$ have to respect the heavy computational load imposed by the setup. Thus, the greedy strategy has become the most popular computational approach for this problem. It chooses consecutively elements $f_n \in \mathcal{F}$ and defines $\mathcal{H}_n := \mathrm{span}\{f_1, ..., f_n\}$ starting with $\mathcal{H}_0 := \{0\}$. Given $\mathcal{H}_n$, the element $f_{n+1} \in \mathcal{F}$ is defined as a maximizer of the distance $\mathrm{dist}(\mathcal{F}, \mathcal{H}_n) := \sigma_n(\mathcal{F})$ for the (pure) greedy algorithm, or any element $f_{n+1} \in \mathcal{F}$ for which $\mathrm{dist}(f_{n+1}, \mathcal{H}_n) \geq \gamma\sigma_n(\mathcal{F})$ with $0 < \gamma \leq 1$ for the weak greedy algorithm. It is natural to compare the approximation performance of the $\mathcal{H}_n$ generated by this strategy with that of the Kolmogorov widths $d_n(\mathcal{F})$ since the latter gives the smallest error that can be achieved by subspaces of fixed dimension n. The first such comparisons, given in [1], show that the approximation error, obtained by the pure greedy strategy satisfies $\sigma_n(\mathcal{F}) \leq Cn2^n d_n(\mathcal{F})$. Here we present various improvements of this result. Among these, it is shown that whenever $d_n(\mathcal{F}) \leq Mn^{-\alpha}$, for all $n > 0$, and some $M, \alpha > 0$, we also have $\sigma_n(\mathcal{F}) \leq C_\alpha M n^{-\alpha}$ for all $n > 0$, where C_α depends only on α. Similar results are derived for exponential rates of the form Me^{-an^α}. These results are further generalized for the computationally feasible variant of the weak greedy algorithm in which instead of f_n one receives its noisy version $\tilde{f}_n$ that is not even necessarily in $\mathcal{F}$. All we know about $\tilde{f}_n$ is that $\mathrm{dist}(f_n, \tilde{f}_n) \leq \varepsilon$. As before, we define $\tilde{\mathcal{H}}_n := \mathrm{span}\{\tilde{f}_1, ..., \tilde{f}_n\}$ and $\tilde{\sigma}_n(\mathcal{F}) := \mathrm{dist}(\mathcal{F}, \tilde{\mathcal{H}}_n)$. The following result shows the robustness of the weak gready algorithm with respect to this perturbation.

Theorem 1. *Suppose that* $d_0(\mathcal{F}) \leq M$ *and*

$$d_n(\mathcal{F}) \leq Mn^{-\alpha}, \qquad \text{alternatively} \quad d_n(\mathcal{F}) \leq Me^{-an^\alpha}, \qquad n > 0,$$

*This research was supported by the Office of Naval Research Contract ONR-N00014-07-1-0978; the NSF Grants DMS-0721621 and DMS-0915104; and the Bulgarian Scientific Foundation grant contract #DO 02-102/23.04.2009.

for some $M, a, \alpha > 0$. Then, setting $\beta := \frac{\alpha}{1+\alpha}$, one has

$$\tilde{\sigma}_n(\mathcal{F}) \leq C \max\{Mn^{-\alpha}, \varepsilon\},$$

$$\textit{alternatively} \quad \tilde{\sigma}_n(\mathcal{F}) \leq C \max\{Me^{-cn^{\beta}}, \varepsilon\}, \qquad n > 0,$$

where c, C are constants that depend on a, α and on the weakness parameter γ.

The main findings and proofs of the results from this talk are provided in [2].

Keywords: Reduced basis method, week greedy approximation, Kolmogorov widths.
AMS Classification: 41A46, 65M60, 41A25, 41A63.

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A bi-orthogonal expansion in the space $L^2(0, \infty)$ *

André Boivin and Changzhong Zhu

Abstract

Assume that a sequence of complex numbers $\{\lambda_k\}$ $(k = 1, 2, \ldots)$ satisfies the conditions: $\Re(\lambda_k) > 0$, $\lambda_k \neq \lambda_j$ for $k \neq j$ and $\sum_{k=1}^{\infty} \frac{\Re(\lambda_k)}{1+|\lambda_k|^2} < +\infty$. It is known that under the above conditions, the Blaschke product $W(\xi) = \prod_{k=1}^{\infty} \left[\frac{\xi-\lambda_k}{\xi+\overline{\lambda}_k} \cdot \frac{|1-\lambda_k^2|}{1-\lambda_k^2} \right]$ converges to an analytic function $W(\xi)$ in the right half-plane $\Re(\xi) > 0$, and that (see [5]) the exponential system

$$\{e^{-\lambda_k x}\} \quad (k = 1, 2, \ldots) \tag{1}$$

is incomplete in $L^2(0, \infty)$. V. Kh. Musoyan [5, p. 65] showed that if

$$\psi_k(x) = -\frac{1}{\overline{W'(\lambda_k)}} \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{e^{-i\tau x}}{W(i\tau)(i\tau + \overline{\lambda}_k)} d\tau \quad (k = 1, 2, \ldots), \tag{2}$$

then the systems (1) and (2) are bi-orthogonal in $L^2(0, +\infty)$. Using the Fourier transform and corresponding results in the Hardy space H_+^2 for the upper half-plane, the bi-orthogonal expansions with respect to the systems (1) and (2) will be obtained.

Keywords: Bi-orthogonal expansion, Hardy space, exponential system.
AMS Classification: 30D55.

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Discrete Kolmogorov inequality for multiply monotone sequences

Yuliya Babenko and Sergiy Borodachov

Abstract

We study multiplicative Kolmogorov type inequalities involving norms of sequences, which are positive together with their differences up to a given order $r-1$ ($(r-1)$-monotone sequences), and norms of their forward differences. The inequalities we deal with estimate the q-norm of the k-th order difference via the p-norm of the sequence and the sup-norm of the r-th order difference ($0 < k < r$). Reviews of known results on inequalities for norms of finite differences can be found in [1] and for norms of derivatives of functions – in [2]. We will present a sharp inequality of the described type for every $0 < k < r$ and certain triplets (p,q,s). Moreover, we obtain a more delicate information and provide the description of the modulus of continuity, which gives the supremum of the q-norm of the k-th difference of an $(r-1)$-monotone sequence provided that the p-norm of the sequence and the sup-norm of its r-th difference are fixed. For certain values of k, p, and q, our results are valid for a wider class of $(r-2)$-monotone sequences.

Keywords: Sequences, finite differences, multiply monotone, discrete inequality, rearrangements, comparison theorem.

AMS Classification: 26D10, 47B39, 39A70, 65L12.

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On orthogonal Laurent polynomials in two variables*

Cleonice F. Bracciali, Lidia Fernández, Teresa E. Pérez and Miguel A. Piñar

Abstract

We introduce the Laurent polynomials in two variables by deducing a lexicographic order for the elements of the basis. We define orthogonal Laurent polynomials in two variables with respect to an strong moment functional. Necessarily and sufficient conditions for the existence of the orthogonal Laurent polynomials and a system of recurrence relations are obtained. Finally, some examples are given.

Keywords: Orthogonal polynomials in two variables, orthogonal Laurent polynomials in two variables.

AMS Classification: 42C05, 33C50.

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*First author thanks CAPES, CNPq and FAPESP from Brazil, others authors thank Micinn of Spain and ERDF through the grant MTM 2008–06689–C02–02, and Junta de Andalucía through the grant FQM 0229.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

The convergence of zero sequences in the indeterminate rational Stieltjes moment problem*

Adhemar Bultheel, Pablo González-Vera, Erik Hendriksen and Olav Njåstad

Abstract

In the strong or two-point Stieltjes moment problem, one has to find a positive measure on $[0, \infty)$ for which infinitely many moments are prescribed at the origin and at infinity. Here we consider a multipoint version in which the origin and the point at infinity are replaced by sequences of points that may or may not coincide. In the indeterminate case, two natural solutions μ_0 and μ_∞ exist that can be constructed by a limiting process of approximating quadrature formulas. The supports of these natural solutions are disjoint (with possible exception of the origin). The support points are accumulation points of sequences of zeros of even and odd indexed orthogonal rational functions. These functions are recursively computed and appear as denominators in approximants of continued fractions. They replace the orthogonal Laurent polynomials that appear in the two-point case. In this paper we consider the properties of these natural solutions and analyse the precise behaviour of which zero sequences converge to which support points. This generalizes results that were obtained for the strong Stieltjes moment problem.

For relevant discussion of the strong Stieltjes moment problem see [1, 2, 3, 8, 9, 10, 11]. For the rational moment problem see [4, 5, 6, 7] and further references found there. In this contribution we shall in particular generalize the results of [3].

Keywords: strong Stieltjes moment problem, orthogonal rational functions, natural solutions, indeterminate moment problem, multipoint approximation, continued fractions.

AMS Classification: 30E05, 44A60, 42C05, 30B70.

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*The work of the first author is partially supported by the Belgian Network DYSCO (Dynamical Systems, Control, and Optimization), funded by the Interuniversity Attraction Poles Programme, initiated by the Belgian State, Science Policy Office. The scientific responsibility rests with the author. The first and the second author are also supported by the Direción General de Investigación, Ministerio de Educación y Ciencia, under grant MTM2008-2011.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Two classical polynomials in hypercomplex function theory*

I. Cação, M. I. Falcão and H. R. Malonek

Abstract

Classical polynomials of a real or complex variable and their generalizations to the case of several real or complex variables are well-known and have widely been studied. We propose a different approach to the higher dimensional case for the construction of analogues of Hermite and Laguerre polynomials in $\mathbb{R}^{n+1}$ by using hypercomplex function theoretic tools. As part of Clifford Analysis, hypercomplex function theory generalizes the theory of holomorphic functions of one complex variable by using Clifford algebras. In this framework the analogue of holomorphic functions is obtained as the set of null-solutions to a generalized Cauchy-Riemann system (by historical reasons also called monogenic functions). Contrary to the complex case, the construction of monogenic polynomials is not a trivial task mainly because neither the canonical hypercomplex variable nor its powers are monogenic. However, a special linear combination involving only products of the canonical hypercomplex variable and its hypercomplex conjugate provides a sequence of monogenic polynomials that form an Appell set with respect to the hypercomplex derivative. Taking that sequence as a basic polynomial sequence associated to some lowering operators, we construct monogenic Hermite and Laguerre polynomials in arbitrary dimensions.

Keywords: Hermite polynomials, Laguerre polynomials, hypercomplex function theory, monogenic polynomials, Appell sequences, exponential operators.

AMS Classification: 30G35, 34L40, 33C45, 33C50.

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Hermite interpolation on the circle and bounded interval. Some results about convergence*

Elías Berriochoa, Alicia Cachafeiro and E. Martínez

Abstract

We study the Hermite interpolation problem on the unit circle for nodal systems more general than those formed by the n-roots of complex numbers with modulus one.

Under suitable assumptions for the nodal system, that is, when it is constituted by the zeros of para-orthogonal polynomials with respect to appropriate measures, we prove the convergence of the Hermite-Fejér interpolation polynomial for continuous functions. Moreover, for the Hermite interpolation problem, we obtain a sufficient condition on the interpolation conditions for the derivatives in order to have uniform convergence for continuous functions.

Finally, we obtain some improvements on the Hermite interpolation problems on the interval, and for the Hermite trigonometric interpolation.

Keywords: Hermite-Fejér interpolation, Hermite interpolation, Laurent polynomials, convergence, unit circle, orthogonal polynomials, para-orthogonal polynomials, Szegő function.

AMS Classification: 33C45, 42C05, 41A05, 41A10.

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*The research was supported by Ministerio de Educación y Ciencia under the grant MTM2008-00341

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New low–storage implementations of time integration Runge Kutta schemes for CAA problems*

J.I. Montijano, L. Rández and M. Calvo

Abstract

In Runge Kutta (RK) time advancing schemes to be used in the semi discretizations of aero acoustics problems by the method of lines, for a give level of accuracy there are two natural requirements to be taken into account in the derivation of efficient integrators: First of all to minimize the storage requirements because of the high dimensionality of this type of problems and secondly maximize the stability properties for the standard wave test equation. For amplification functions of several orders with good stability properties some two-stage RK schemes have been proposed in the For amplification functions of several orders with good stability properties some two–stage RK schemes have been proposed in the last years, see e.g. references [1, 2, 3]. In particular D.I. Ketcheson [3] has derived new families of two–storage schemes that have a larger number of free parameters than the classical families of [1] and [2] and this additional freedom has been used to improved different features of the schemes. In the present paper a new minimum storage family with the same number of parameters as in Ketcheson approach is proposed and the selection of the available parameters with different stability and accuracy requirements is considered.

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*This work was supported by project BFM2001-2562

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Second order differential operators associated with Kantorovich-type operators

Francesco Altomare, Mirella Cappelletti Montano and Vita Leonessa

Abstract

Let K be a convex compact subset K of $\mathbb{R}^p$, $p \geq 1$ and denote by $C(K)$ (resp., $C^2(K)$) the space of all continuous functions on K (resp., the space of all continuous functions on K which are twice continuously differentiable in the interior of K and whose partial derivatives up to the order 2 can be continuously extended on K).

In this talk, we focus on elliptic second order differential operators of the form

$$V(u)(x) := \frac{1}{2}\sum_{i,j=1}^{p} \alpha_{i,j}(x)\frac{\partial^2 u(x)}{\partial x_i \partial x_j} + \sum_{i=1}^{p} \beta_i(x)\frac{\partial u(x)}{\partial x_i} \qquad (u \in C^2(K),\ x \in K), \tag{1}$$

where, for every $i, j = 1, \ldots, N$, the coefficients $\alpha_{i,j}$ depend on a fixed projection $T : C(K) \longrightarrow C(K)$ and the coefficients β_i depend on a sequence $(\mu_n)_{n\geq 1}$ of probability Borel measures on K. As a matter of fact, this class of operators plays an important role in many differential problems arising from genetics, physics, financial mathematics and other fields.

In [2] we have shown that the operator $(V, C^2(K))$ is closable and its closure generates a Feller semigroup $(T(t))_{t\geq 0}$ which in turn is representable as a limit of suitable powers of a particular sequence $(C_n)_{n\geq 1}$ of positive linear operators, i.e.

$$\lim_{n\to+\infty} C_n^{[nt]}(f) = T(t)(f) \tag{2}$$

for every $f \in C(K)$, where $[nt]$ denotes the integer part of nt.

The construction of the sequence $(C_n)_{n\geq 1}$ is based on the projection T and the sequence $(\mu_n)_{n\geq 1}$ and it lies on a generalization of an idea first developed in [1]. We note that $(C_n)_{n\geq 1}$ is a positive approximation process in $C(K)$ and it is connected to the operator V via an asymptotic formula.

Formula (2) implies, in particular, that it is possible to give an explicit representation, in terms of the C_n's, of the solutions to suitable diffusion problems associated with V and also to infer some regularity properties of such solutions by means of similar preservation properties shared by the C_n's themselves.

Keywords: Positive projection, positive approximation process, Kantorovich operator, degenerate second-order elliptic differential operator, Feller semigroup, approximation of semigroup.

AMS Classification: 41A36, 41A25, 41A63, 47D06, 47F05, 35K65.

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• I Jaen Conference on Approximation Theory
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Hyperspectral imaging

Charles Chui

Abstract

Since human vision is limited to electromagnetic radiation in the frequency band of 400 790 terahertz (1 THz = 10^{12} cycles per second), visible light to the human eye is restricted to wavelengths in the range of 400 700 nm (1 meter = 10^9 nanometers). On the other hand, with 12 color channels, the eyes of mantis shrimp have the capability of viewing electromagnetic radiation ranging from ultraviolet (UV: 10 nm 400 nm) to infrared (Near IR: 700 nm 1,000 nm). In other words, mantis shrimp has the hyperspectral vision. With the recent rapid advances of satellite, senor, and computing technologies, it is now possible to capture and analyze hyperspectral image (HSI) data. We will discuss the significance of hyperspectral imaging for a broad spectrum of applications: from agricultural monitoring to geospatial mapping, and from security surveillance to cancerous tissue detection in medical imaging. However, the mathematics of HSI is most challenging and requires introduction of innovative ideas, since the amount of data information in a typical HSI cube is huge. For example, for a 1 megapixel-resolution HSI with 200 spectral bands, the data kernel is a $10^6 \times 200$ matrix. Hence, a very important problem is to reduce the kernel size while preserving the useful data information. This problem is called dimensionality reduction. However, it is well known that linear methods such as principal component analysis (PCA) and multi-dimensional scaling (MDS) are not effective for the study of the problem of dimensionality reduction. I will discuss the general architecture of the current non-linear approaches, the various difficulties that arise, and the defects of current solutions, particularly in neighborhood selection and data set tiling.

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Uniform approximation by means of radical functions

E. Corbacho

Abstract

In this talk we will see how the odd index radical functions can be used to approximating any continuous function on $[a, b]$.

Theorem 1. *Let f be a continuous function defined on $[a, b]$ and let $P = \{x_0 = a,\ x_1, ..., x_n = b\}$ be a partition of $[a, b]$ with $x_i = x_0 + \frac{b-a}{n} \cdot i,\ i = 0, ..., n$. Then, the sequence*

$$\left(f(a) + \sum_{j=2}^{n} [f(x_j) - f(x_{j-1})] \cdot \frac{\sqrt[2n+1]{x_{j-1} - a} + \sqrt[2n+1]{x - x_{j-1}}}{\sqrt[2n+1]{b - x_{j-1}} + \sqrt[2n+1]{x_{j-1} - a}} \right)_n$$

converges uniformly as $n \to \infty$ to f on $[a, b]$.

In addition, the values $k_1 = f(a)$, $k_j = f(x_j) - f(x_{j-1})$, $j = 2, ..., n$ we can change them for other distinct and the previous result is the same.We will illustrate this approximation procedure with a large number of examples.

Keywords: Approximation Theory, radical functions, uniform convergence.

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Hahn's classical multiple orthogonal polynomials

Amílcar Branquinho, Luís Cotrim and A. Foulquié Moreno

Abstract

In this talk, we present an algebraic theory of multiple orthogonal polynomials. Our departure point is the three term recurrence relation, with matrix coefficients, satisfed by a sequence of vectors polynomials. We give some characterization of a Hahn's classical type I and II multiple orthogonal polynomials in terms of a Pearson vector functional equation and structure relations in multiple orthogonality. Comparasions with the cases studied from the work of A.I. Aptekarev, A. Branquinho and W. Van Assche, Multiple orthogonal polynomials for classical weights, Trans. Amer. Math. Soc. **335**, no. 10 (2003), 3887-3914, will also be presented.

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• I Jaen Conference on Approximation Theory
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Multivariate polynomials and their representation in hypercomplex function theory*

C. Cruz and H. R. Malonek

Abstract

Hypercomplex Function Theory is the core of a generalization of Complex Analysis to higher dimensions by studying Clifford-algebra valued functions of one or more non-commutative hypercomplex variables (Clifford Analysis). Its applications, in general, depend from the determination of suitable polynomial bases. In this context and based on a general construction principle not used so far in Clifford Analysis, new representations of multivariate polynomials are presented.

AMS Classification: 30G35, 05A19, 11B68.

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*Financial support from the research unit "CIDMA-Centro de Investigaccão e Desenvolvimento em Matemática e Aplicações" of the University of Aveiro, through the Portuguese Foundation for Science and Technology (FCT), gratefully acknowledged.

Adaptive multidimensional modelling with applications

Annie Cuyt and Oliver Salazar Celis

Abstract

We present a new approximation technique in which a generalized rational model

$$r_{n,m}(x_1,\ldots,x_d) = \frac{\sum_{k=0}^{n} a_k g_k(x_1,\ldots,x_d)}{\sum_{k=0}^{m} b_k g_k(x_1,\ldots,x_d)}$$

is fitted through some interval data $F^{(\ell)} = [f_<^{(\ell)}, f_>^{(\ell)}], \ell = 0,\ldots,s$:

$$r_{n,m}\left(x_1^{(\ell)},\ldots,x_d^{(\ell)}\right) \in F^{(\ell)}.$$

Rational functions offer the possibility to model asymptotic behaviour easily.

The advantage of working with interval data instead of point data is that the interpolation problem inherently takes the data errors, stemming from measurements or simulations, into consideration. In addition, the problem as posed does not suffer the curse of dimensionality: we fit a set of interval values associated with data points in multidimensional space. These data points can appear in a structured way or can be scattered. They are also allowed to be structured in some of the variables and scattered in the others.

Under the condition that the solution set for the coefficients $a_0,\ldots,a_n$, $b_0,\ldots,b_m$ has nonempty interior, the interval fitting problem can be reformulated as a quadratic programming problem.

We illustrate all of the above with applications in a variety of scientific computing problems from video signal filtering, bidirectional reflectance distribution function approximation, network problems, queueing problems, metamodelling in microwave theory and materials science, computational finance, ...

The dimensionality in the examples does not exceed 10. The basis functions are either orthogonal polynomials or trigonometric functions. The number of data can run up to a few million. In all of the applications a model $r_{n,m}$ of sufficiently low complexity could be computed: $n + m \leq 100$.

Keywords: Interval, interpolation, multivariate, rational, scientific computing, error control, modelling.

AMS Classification: 41A20, 41A05, 65D05.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Computing multivariate Fekete and Leja points by numerical linear algebra*

Len Bos, Stefano De Marchi, Alvise Sommariva and Marco Vianello

Abstract

We present two greedy algorithms, that compute discrete versions of Fekete-like points for multivariate compact sets by basic tools of numerical linear algebra. The first gives the so-called *Approximate Fekete Points* by QR factorization with column pivoting of Vandermonde-like matrices. The second computes *Discrete Leja Points* by LU factorization with row pivoting.

Indeed, in some recent papers, a simple and effective greedy algorithm to extract approximate Fekete points from admissible meshes has been studied, and succesfully applied in various instances, cf. [8, 9, 29, 30]. The algorithm gives an approximate solution to a nonlinear combinatorial optimization problem (discrete maximization of the Vandermonde determinant) using only a basic tool of numerical linear algebra, namely the QR factorization with column pivoting.

Here we also pursue an alternative greedy algorithm for discrete maximization on (weakly) admissible meshes, i.e., the computation of the so-called Leja points of the mesh. Following an idea recently proposed by R. Schaback [27], we show that this algorithm can be easily implemented by another basic tool of linear algebra, the LU factorization with partial (row) pivoting. We recall that approximate Fekete points, computed with any basis from a weakly admissible mesh, are asymptotically equidistributed with respect to the pluripotential-theoretic equilibrium measure of the compact set; cf. [8]. Here we prove that the same is true for discrete Leja points computed with a special class of polynomial bases.

Finally, we discuss the asymptotic distribution of such points when they are extracted from *Weakly Admissible Meshes*. These meshes are nearly optimal for least-squares approximation and contain interpolation sets nearly as good as Fekete points of the domain. These allow us to replace a continuous compact set by a discrete version, that is "just as good" for all practical purposes. Some examples will be presented in order to show the effectiveness of the algorithms.

*This work has been supported by the "ex-60%" funds and by the project "Interpolation and Extrapolation: new algorithms and applications" of the University of Padova, and by the INdAM GNCS.

Keywords: Weakly admissible meshes, approximate Fekete points, discrete Leja points, Vandermonde matrices, QR factorization with column pivoting, LU factorization with row pivoting, pluripotential theory, equilibrium measure.

AMS Classification: 41A10, 41A63, 65D05.

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A new characterization of the Lorentz spaces $L(p, 1)$ for $1 < p < \infty$ and applications

Geraldo Soares de Souza

Abstract

In 1950, **G. G. Lorentz** introduced in his paper entitled "**Some New Functional Spaces**" at Annals of Mathematics, the function spaces denoted by $\Lambda(\alpha)$ for $0 < \alpha < 1$, defined as the set of real measurable functions $f(x), 0 < x < 1$ for which

$$||f||_{\Lambda(\alpha)} = \alpha \int_0^1 x^{\alpha-1} f^\star(x) dx,$$

where $f^\star$ is the decreasing rearrangement of f.
In this talk we give two simple characterizations of the special atoms space of $\Lambda\left(\frac{1}{p}\right)$ for $1 < p < \infty$ based on a generalizations of the special atoms spaces, introduced by **Geraldo De Souza** in his earlier works. The space $\Lambda\left(\frac{1}{p}\right)$ is nowadays denoted by $L(p, 1)$.
We use these characterizations to give a rather simple proof of **Weiss-Stein** theorem on the extension of operators on $L(p, 1)$ and also we take a look at Carleson Theorem on Convergence of Fourier series for this space.

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Asymptotic behavior of complex orthogonal polynomials related with Gaussian quadrature

A. Deaño, D. Huybrechs and A.B.J. Kuijlaars

Abstract

We present results on the asymptotic behavior of a family of polynomials which are orthogonal with respect to an exponential weight on certain contours of the complex plane. Our motivation comes from the fact that the zeros of these polynomials are the nodes for complex Gaussian quadrature of an oscillatory integral defined on the real axis and having a high order stationary point. The limit distribution of these zeros is also analyzed, and we show that they accumulate along a contour in the complex plane that has the S-property in the presence of an external field. Additionally, the strong asymptotics of the orthogonal polynomials is obtained by applying the nonlinear Deift–Zhou steepest descent method to the corresponding Riemann–Hilbert problem.

Keywords: Riemann–Hilbert problems, complex orthogonal polynomials, Gaussian quadrature, asymptotic analysis.

AMS Classification: 35Q15, 65D32, 42C05, 34E05.

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On the rate of approximation of convolution operators in homogeneous Banach spaces on $\mathbb{R}^d$ or $\mathbb{T}^d$*

Borislav R. Draganov

Abstract

We present a method for establishing direct and strong converse inequalities in terms of K-functionals for convolution operators acting in homogeneous Banach spaces on $\mathbb{R}^d$ or $\mathbb{T}^d$. These spaces are a natural generalization of the L_p-spaces with $1 \le p < \infty$ and the spaces of uniformly continuous and bounded functions. The method is based on the behaviour of the Fourier transform of the kernel of the convolution operator. The differential operator in the K-functional can be defined by the convolution operator similarly to infinitesimal generator and described explicitly by means of the Fourier transform. This description turns out to be very simple in various important cases and identifies the differential operator as some well-known such operators among which the strong Riesz fractional derivative. Saturation of convolution operators is also considered. Part of the results has recently appeared in *Journal of Approximation Theory* **162** (2010), 952–979.

Keywords: Convolution operator, singular integral, trigonometric polynomial, rate of convergence, degree of approximation, saturation, K-functional, homogeneous Banach space, Fourier-Stieltjes transform, Fourier series, multipliers.

AMS Classification: 41A25, 41A27, 41A35, 41A36, 41A40, 41A63, 41A65, 42A10, 42A38, 42A45, 42A50, 42A85, 42B10, 42B20.

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*Supported by the University of Sofia and the European Operational Programme HRD through contract BGO051PO001/07/3.3-02/53 with the Bulgarian Ministry of Education.

Rational Gauss-type quadrature formulas with a prescribed node anywhere on the real line*

Karl Deckers, Adhemar Bultheel and Joris Van Deun

Abstract

In general, the zeros of an orthogonal rational function (ORF) on a subset of the real line, with poles among $\{\alpha_1, \ldots, \alpha_n\} \subset (\mathbb{C}_0 \cup \{\infty\})$, are not all real (unless α_n is real), and hence, they are not suitable to construct a rational interpolatory quadrature rule with positive weights and maximal domain of validity. For this reason, the zeros of a so-called quasi-ORF are used instead. These zeros depend on one single parameter $\tau \in (\mathbb{C} \cup \{\infty\})$, which can be chosen freely in order to fix one node in the rational quadrature rule. First, we give a condition on the parameter τ to ensure that the zeros of the quasi-ORF are all real and simple. Next, we provide a generalized eigenvalue problem to compute the zeros of the quasi-ORF and the corresponding weights in the rational quadrature rule.

Keywords: Rational Gauss-type quadrature, quasi-orthogonal rational functions, generalized eigenvalue problem.

AMS Classification: 42C05, 65D32, 65F15.

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*The first author is a Postdoctoral Fellow of the Research Foundation - Flanders (FWO).
This work is partially supported by the Belgian Network DYSCO (Dynamical Systems, Control, and Optimization), funded by the Interuniversity Attraction Poles Programme, initiated by the Belgian State, Science Policy Office. The scientific responsibility rests with its authors.

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Krall–type othogonal polynomials on the unit ball

A. M. Delgado, L. Fernández, T. E. Pérez and M. A. Piñar

Abstract

We present a modification of the two variable classical measure on the ball, by adding a finite set of equally spaced mass points on the border. In such a case, both the orthogonal polynomials and the reproducing kernels associated with this measure can be explicitely expressed in terms of those corresponding with the classical measure. Then, the asymptotics of the kernels and the Christoffel functions are studied.

Keywords: Two variable orthogonal polynomials, asymptotics.
AMS Classification: 42C05, 33C50.

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On the extension of the Shepard-Bernoulli operators to higher dimensions

R. Caira, F.A. Costabile, F. Dell'Accio and F. Di Tommaso

Abstract

We present special extensions [5] to the bivariate case of the Shepard-Bernoulli opera- tors introduced in [1] and of several others univariate combined Shepard operators [2]. These new interpolation operators are realized by using bivariate three point extensions of univariate expansions formulas (see, for example, [3, 4]) in combination with bivariate Shepard operators [6] and do not require special partitions of the node convex hull. We study properties of the new operators and provide applications to the scattered data interpolation problem.

Keywords: Multivariate polynomial interpolation, degree of exactness, scattered data interpolation, combined Shepard operator, modified Shepard operator.

AMS Classification: (primary) 41A05, 41A25; (secondary) 65D05, 65D1.

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Zero distribution of matrix and multiple orthogonal polynomials*

Steven Delvaux

Abstract

We consider matrix and multiple orthogonal polynomials on the real line in case where the recurrence coefficients are asymptotically periodic, slowly varying with n, or both. Under these assumptions, we obtain ratio asymptotics for the polynomials and characterize their asymptotic zero distribution. In this way we generalize some recent results in the literature. An important role in our method is played by an algebraic equation related to a (locally) block Toeplitz matrix.

Keywords: Matrix orthogonal polynomial, multiple orthogonal polynomial, recurrence coefficient, Nevai class, ratio asymptotics, asymptotic zero distribution.

AMS Classification: 42C05.

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Adaptive sampling recovery based on quasi-interpolant representations*

Đinh Dũng

Abstract

We propose an approach to study optimal methods of adaptive sampling recovery of functions by sets of a finite capacity which is measured by their cardinality or pseudo-dimension. Let $W \subset L_q$, $0 < q \le \infty$, be a class of functions on $[0,1]^d$. For B a subset in L_q, we define a sampling recovery method with the free choice of sample points and recovering functions from B as follows. For each $f \in W$ we choose n sample points. This choice defines n sampled values. Based on these sampled values, we choose a function from B for recovering f. The choice of n sample points and a recovering function from B for each $f \in W$ defines a sampling recovery method S_n^B by functions in B. An efficient sampling recovery method should be adaptive to f.

Given a family $\mathcal{B}$ of subsets in L_q, we consider optimal methods of adaptive sampling recovery of functions in W by B from $\mathcal{B}$ in terms of the quantity

$$R_n(W,\mathcal{B})_q := \inf_{B \in \mathcal{B}} \sup_{f \in W} \inf_{S_n^B} \|f - S_n^B(f)\|_q.$$

Denote $R_n(W,\mathcal{B})_q$ by $e_n(W)_q$ if $\mathcal{B}$ is the family of all subsets B of L_q such that the cardinality of B does not exceed 2^n, and by $r_n(W)_q$ if $\mathcal{B}$ is the family of all subsets B in L_q of pseudo-dimension at most n. Let $0 < p, q, \theta \le \infty$ and α satisfy one of the following conditions:

(i) $\alpha > d/p$;

(ii) $\alpha = d/p$, $\theta \le \min(1,q)$, $p,q < \infty$.

Then for the d-variable Besov class $U^\alpha_{p,\theta}$ (defined as the unit ball of the Besov space $B^\alpha_{p,\theta}$), there is the following asymptotic order

$$e_n(U^\alpha_{p,\theta})_q \asymp r_n(U^\alpha_{p,\theta})_q \asymp n^{-\alpha/d}.$$

*This work is supported by National Foundation for Science and Technology Development

In comparing with the well-known asymptotic order of optimal non-adaptive sampling recovery

$$n^{-\alpha/d+(1/p-1/q)_+},$$

for $p < q$, the convergence rate of optimal adaptive sampling recovery method is better than that of any non-adaptive (non-linear) one.

To construct asymptotically optimal adaptive sampling recovery methods for $e_n(U^\alpha_{p,\theta})_q$ and $r_n(U^\alpha_{p,\theta})_q$ we use a quasi-interpolant wavelet representation of functions in Besov spaces associated with some equivalent discrete quasi-norm.

The main results of this talk are published in [1] and [2].

Keywords: Adaptive sampling recovery, quasi-interpolation wavelet representation, B-spline, Besov space.

AMS Classification: 41A46, 41A05, 41A25, 42A40.

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Elliptic operators, Feller semigroups and modified Bernstein-Schnabl operators

F. Altomare, M. Cappelletti Montano and S. Diomede

Abstract

We investigate a class of elliptic second-order differential operators acting on compact convex subsets of some euclidean space, and we prove that the closures of such operators generate Feller semigroups.

Moreover, we show that the semigroups may be constructively approximated by iterates of the so-called modified Bernstein-Schnabl operators, which we introduce in [1]. That approximation allows us to explicitely represent the solutions to the abstract Cauchy problems governed by the differential operators and also to infer some regularity properties of such solutions; to this regard we provide a concrete application arising from genetics.

Furthermore, in [2] we discuss the more general case in which the differential operators act on some metrizable convex and compact subset of a locally convex space.

Keywords: Degenerate elliptic differential operator, generation of positive operator semigroup, approximation of semigroup.

AMS Classification: 47D06, 41A36, 41A65, 35K65.

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Interlacing of zeros of Gegenbauer polynomials of non-adjacent degree from different sequences

Kathy Driver

Abstract

We study the interlacing property of zeros of Gegenbauer polynomials C_n^λ of non-adjacent degree from sequences corresponding to different values of the parameter λ where $\lambda > -\frac{1}{2}$. Our results extend Stieltjes' classical result that the zeros of orthogonal polynomials of non-adjacent degree satisfy interlacing properties. We also determine the polynomials that play an analogous role to the de Boor-Saff polynomials in completing the interlacing picture for the zeros of the Gegenbauer polynomial C_{n+1}^λ with the zeros of $C_{n-k}^{\lambda+k}$, $k = 1, \ldots, n-1$.

Keywords: Gegenbauer Polynomials, ultraspherical polynomials, interlacing zeros, construction of orthogonal sequences.

AMS Classification: 33C45, 42C05.

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Optimally space localized polynomials in some classical settings

Wolfgang Erb

Abstract

In this talk, we investigate the space localization of polynomials in some classical settings, i.e. in the case that the polynomials have expansions in terms of Hermite, Gegenbauer or Jacobi polynomials. Therefore, we consider certain well-known uncertainty principles in these settings and the corresponding formulas for the space variance. Then, we determine those polynomials of a fixed degree n that minimize the term for the space variance, i.e. those polynomials that are best localised in space with respect to the given uncertainty principle. Finally, we show how these optimally space localized polynomials can be used as approximation kernels on the unit circle, the n-sphere or on projective spaces and we investigate some of their properties.

Keywords: Uncertainty principles, space localization of polynomials.

AMS Classification: 33C50, 42C05.

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On wavelets and frames in dyadic harmonic analysis

Yu. A. Farkov

Abstract

Orthogonal wavelets and refinable functions representable as lacunary Walsh series have been initiated in [4]; recent results in this direction can be found in [1, 2, 3] and the references therein. Let $\mathbb{R}_+$ be the positive half-line with dyadic addition (see [5, p.421]). We will discuss:

- an algorithm for constructing compactly supported orthogonal wavelets on $\mathbb{R}_+$ associated with the Walsh polynomials;
- necessary and sufficient conditions for a refinable function to generate a multiresolution analysis in $L^2(\mathbb{R}_+)$;
- estimates of the smoothness of dyadic orthogonal wavelets of Daubechies type;
- examples of frames for $L^2(\mathbb{R}_+)$.

Similar results can be obtained in a more general setting, e.g., for biorthogonal wavelets on the Cantor dyadic group (cf. [3, 4]).

Keywords: Walsh functions, Walsh-Dirichlet kernel, Cantor dyadic group, Vilenkin groups, smoothness, orthogonal wavelets, multiresolution analysis, frames.

AMS Classification: 42A55, 42C10, 42C40, 43A70.

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Approximation by polynomial solutions of elliptic equations*

Konstantin Yu. Fedorovskiy

Abstract

Let L be a homogeneous elliptic differential operator in $\mathbb{C}$ with constant complex coefficients. A function f is called L-analytic on an open set $U \subset \mathbb{C}$ if $Lf = 0$ in U; a polynomial P is called L-polynomial if $LP \equiv 0$. Let X be a compact set in $\mathbb{C}$, X° be its interior and $m \geq 1$ be an integer. We consider the following problems:

(1) What conditions on X are necessary and sufficient in order that each function which is continuous on X and L-analytic on X° can be uniformly on X approximated by L-polynomials, and

(2) What conditions on X are necessary and sufficient in order that each function f which is of the class C^m in a neighborhood of X and L-analytic on X° can be approximated by some sequence (P_n) of L-polynomials so that $P_n \to f$ and $\nabla^k P_n \to \nabla^k f$ uniformly on X as $n \to \infty$ and $k = 1, \dots, m$?

In the talk it is planned to discuss the state of the art, some recent results (see [1, 2, 3, 5, 6, 7]) and open questions in these themes. In particular in the case of problem (2) for $m = r - 1$, where r is the order of L, the necessary and sufficient approximability condition is that the set $\mathbb{C} \setminus X$ is connected. This result may be considered as a natural analogue of the classical Mergelyan's theorem on uniform approximation by complex polynomials. In the case of problem (1) the special attention will be given to the case when $L = \overline{\partial}^n$ is the n-th power of the Cauchy-Riemann operator (in this case one deals with approximation by *polyanalytic polynomials*).

If time permits, the rest of the talk will be devoted to the concept of a *Nevanlinna domain* (see [2, Definition 2.1]), which is the special analytic characteristic of a planar domain that have naturally appeared in problems on uniform approximation by polyanalytic polynomials. It is planned to discuss several analytical and geometrical properties of Nevanlinna domains and some open problems related with this concept (see [2, 4, 6]).

Keywords: uniform approximation, C^m-approximation, polynomial solutions of elliptic equations, polyanalytic functions and polyanalytic polynomials.

AMS Classification: 30E10, 35A35.

*The author is partially supported by the Russian Foundation for Basic Research (grants no 09-01-12160-ofi_m) and by the program "Leading Scientific Schools" (grant no. NSh-3476.2010.1).

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A Cohen type inequality for Fourier expansions of orthogonal polynomials with a non-discrete Jacobi-Sobolev inner product

Bujar Xh. Fejzullahu and Francisco Marcellán

Abstract

In this contribution we deal with Fourier expansions in terms of sequences of polynomials $(Q_n^{(\alpha,\beta)}(x))_{n\geq 0}$ orthogonal with respect to the Sobolev inner product

$$\langle f,g\rangle = \int_{-1}^{1} f(x)g(x)d\mu_{\alpha,\beta}(x) + \lambda \int_{-1}^{1} f'(x)g'(x)d\mu_{\alpha+1,\beta}(x)$$

where $\lambda > 0$ and $d\mu_{\alpha,\beta}(x) = (1-x)^{\alpha}(1+x)^{\beta}dx$ with $\alpha > -1$, $\beta > -1$. Notice that this is not a standard inner product, that is $\langle xf,g\rangle \neq \langle f,xg\rangle$. Therefore, nice properties of the standard orthogonal polynomials, such as the three-term recurrence relation, do not hold any more. As a consequence, Christoffel-Darboux formula does not work at all.

On the other hand, the linear space

$$W_p^{\alpha,\beta} = \{f \ : \ \|f\|^p_{W_p^{\alpha,\beta}} = \int_{-1}^{1} |f(x)|^p d\mu_{\alpha,\beta}(x) + \lambda \int_{-1}^{1} |f'(x)|^p d\mu_{\alpha+1,\beta}(x) < \infty\}$$

for $p = 2$ is a Hilbert space and the linear space of polynomials is a dense subset. Thus, $||S_n(f)||_{W_2^{\alpha,\beta}} \leq c||f||_{W_2^{\alpha,\beta}}$ for any function $f \in W_2^{\alpha,\beta}$, where by $S_n(f)$ we denote the nth partial sum of the Fourier expansion of f in terms of our Sobolev orthogonal polynomials. Then, a natural question concerning the existence of $p \neq 2$ such that $||S_n(f)||_{W_p^{\alpha,\beta}} \leq c||f||_{W_p^{\alpha,\beta}}$ for any function $f \in W_p^{\alpha,\beta}$ arises.

In this talk we analyze this problem as well as we prove a Cohen type inequality for such a Fourier expansion. Necessary conditions for its norm convergence are given. Finally, the failure of the a.e. convergence of the Fourier expansion of a function of $W_p^{\alpha,\beta}$ in terms of such an orthogonal polynomial family is deduced.

Keywords: Jacobi-Sobolev orthogonal polynomials, Fourier expansions, Cohen type inequality.

AMS Classification: 42C05, 42C10.

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• I Jaen Conference on Approximation Theory
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The Uvarov modification of simplex polynomials

A. M. Delgado, L. Fernández, T. E. Pérez and M. A. Piñar

Abstract

We consider the classical Jacobi measure on the simplex with additional masses at the vertices. Orthogonal polynomials in several variables associated with this measure can be explicitly expressed in terms of orthogonal polynomials on the simplex, so are the reproducing kernels associated with these polynomials. Using these explicit expressions the asymptotic behavior of the kernels is obtained.

Keywords: Multivariate orthogonal polynomials, asymptotics.
AMS Classification: 42C05, 33C50.

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A piecewise polynomial approach to analyzing interpolatory subdivision

Michael S. Floater

Abstract

The four-point interpolatory subdivision scheme of Dubuc and its generalizations to irregularly spaced data studied by Warren and by Daubechies, Guskov, and Sweldens are based on fitting cubic polynomials locally. In this talk we analyze the convergence of the scheme by viewing the limit function as the limit of piecewise cubic functions arising from the scheme. This allows us to recover the regularity results of Daubechies et al. in a simpler way and to obtain the approximation order of the scheme and its first derivative.

Keywords: Interpolatory subdivision, piecewise polynomials.

AMS Classification: 65D05, 65D17.

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Perturbations on the subdiagonals of Toeplitz matrices

K. Castillo, L. Garza and F. Marcellán

Abstract

Let $\mathcal{L}$ be an Hermitian linear functional defined on the linear space of Laurent polynomials. It is very well known that the Gram matrix of the associated bilinear functional in the linear space of polynomials is a Toeplitz matrix. In this contribution we analyze some linear spectral transforms of $\mathcal{L}$ such that the corresponding Toeplitz matrix is the result of the addition of a constant in a subdiagonal of the initial Toeplitz matrix. We focus our attention in the analysis of the quasi-definite character of the perturbed linear functional as well as in the explicit expressions of the new monic orthogonal polynomial sequence in terms of the first one.

Keywords: Hermitian linear functionals, Caratheodory functions, Laurent polynomials, orthogonal polynomials, Toeplitz matrices.

AMS Classification: 42C05, 15A23.

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An extremal property for a class of discretely defined positive linear operators

Ioan Gavrea

Abstract

J. M. Aldaz and H. Render, [1], show that a certain optimality property of the classical Bernstein operator also holds for generalized Bernstein operators on extended Chebysev systems. Using a similar technique like in [2], we show that this optimality property is not specified for generalized Bernstein operators; it takes place for a wide class of discretely defined positive linear operators.

Keywords: Optimality property, positive linear operators, generalized Bernstein operators.

AMS Classification: 41A35, 41A50.

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Orthogonal polynomials and Fourier coefficients for bivariate Szego-Bernstein measures

Jeffrey Geronimo

Abstract

I will discuss some recent results concerning orthogonal polynomials and Fourier coefficients associated with Szego-Bernstein measures supported on the bicircle.

Keywords: Orthogonal polynomials, bircircle, Szego-Bernstein measures.

AMS Classification: 42C05, 30E05, 47A57.

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Online learning with non-identical sampling processes

Ting Hu and Ding-Xuan Zhou

Abstract

Online learning is an important family of algorithms in learning theory and widely used for many practical applications. Its main advantage is the low complexity of computation. Here we consider online algorithms based on kernel methods and conduct error analysis by approximation in reproducing kernel spaces.

Classical learning is often done in the setting that the samples are drawn independently from an identical distribution (i.i.d.). Such conditions are always not satisfied for real situations. Here we study the online learning algorithms with samples independently drawn from a non-identical sequence of probability distributions. Error bounds are given in terms of the approximation properties of target functions and the parameters of the learning algorithms.

We also consider online learning by Gaussians. The variance of a Gaussian kernel is a measurement of the frequency range of function components or features retrieved by learning algorithms induced by the Gaussian. The approximation power increase when the variance of the Gaussian decreases. Thus it is natural to use Gaussians with decreasing variances for online algorithms when samples are imposed one by one.

Keywords: Learning theory, Gaussian kernel, variance of Gaussian, online learning, classification algorithm, convex loss function, reproducing kernel Hilbert space, regression problems.

AMS Classification: 68Q32, 68T05, 62J02, 62L20.

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Zeros of orthogonal polynomials generated by canonical perturbations of the standar measure*

Edmundo J. Huertas, Francisco Marcellán and Fernando R. Rafaeli

Abstract

In the last years some attention has been paid to the so called canonical spectral transformations of measure supported on the real line. Our contribution is focused on the behaviour of zeros of MOPS associated with the Christoffel and Uvarov transformations of such measures.

The outline of the talk is the following. In the first part we introduce the representation of the perturbed MOPS in terms of the initial ones and we analyze the behaviour of the zeros of the MOPS when an Uvarov transform is introduced. In particular, we obtain such a behavior when the mass N tends to infinity as well as we characterize the values of the mass N such the smallest (respectively, the largest) zero of these MOPS is located outside the support of the measure. The second part of the talk is devoted to the electrostatic interpretation of the zero distribution as equilibrium points in a logarithmic potential interaction under the action of an external field. We analyze such an equilibrium problem when the mass point is located either in the boundary or in the exterior of the support of the measure, respectively.

Keywords: Orthogonal polynomials, zeros of polynomials, Christoffel transforms, Uvarov transforms, connection formula, structure relation, electrostatic interpretation.

AMS Classification: 33C47.

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*The work of the first and second authors has been supported by Dirección General de Investigación, Ministerio de Ciencia e Innovación of Spain, grant MTM2009-12740-C03-01.
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Biorthogonal polynomials and a conjecture by Adler and van Moerbecke

A. Ibort, J. Arvesú and C. Esposito

Abstract

We will discuss a conjecture by Adler and van Moerbecke [1] on the existence of generalized sequences of weights for families of polymomials satisfying generalized recursion relations [3]. We will determine the solution to the corresponding problem of moments [2] and we will discuss its relation with biorthogonal polynomials and multiple orthogonal polynomials. The spectral theory of the corresponding Jacobi operators and their self–adjoint extensions [4] will be analized.

Keywords: Biorthogonal polynomials, multiple orthogonal polynomials, generalized recurrence relations, self–adjoint extensions.

AMS Classification: 41A28, 41A40, 41A60.

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On convergence and calculation of rational quadrature rules*

J. Illán

Abstract

The starting point of our work is the numerical evaluation of an integral as that shown below

$$Q(f, W) = \int_I f(x)W(x)dx,$$

where f is analytic on $I = [a, b]$ and meromorphic in $V \supset [a, b]$. Moreover, W is a non-negative weight function with integrable singularities at the endpoints of I.

To carry out this task we use a quadrature rule given in terms of a modification BW/A, where A and B are polynomials. The design to be followed should mitigate the adverse effect produced by nearby poles and zeros of f, when a standard procedure is applied. Firstly, we multiply W by a factor B_0/A_0, where B_0 and A_0 are positive polynomials whose respective roots simulate zeros and poles of f.(1 This close relationship between the integrand and these polynomials A_0 and B_0, should raise the computational cost. This approach includes the use of variant weights and the corresponding link to the approximation and interpolation of rational functions. It means that they are also considered two sequences of positive polynomials $\{B_n\}$ and $\{A_n\}$ as part of the final weight, namely, $W_n = B_0B_nW/A_0A_n$.

Let $Q_n^G(f, W_n) = \sum_{k=0}^{n} \lambda_{n,k} f(x_{n,k})$ be the Gauss quadrature rule with respect to W_n. In the first part of our talk we try to give a partial answer to the question: what is the role played by the polynomials A_0, B_0, A_n and B_n, in the asymptotical behavior of the error $|Q(f, W) - Q_n^G(f, W_n)|$. A feature of this procedure is that $\{B_n\}$ and $\{A_n\}$ do not depend on f and, moreover, they are independent of each other. One conclusion is that the contribution of B_n is not always visible.

The so-called B/A-approach, was already introduced by the author in [1], when dealing with interpolatory rules with respect to B_nW/A_n. Once we have modified the integrator we face a non-trivial problem: the calculation of the nodes and coefficients associated with

*The research of J.R.I.G. is supported by a research grant from the Ministry of Science and Innovation, Spain, project code MTM 2008-00341.

[1]In general, we say that a polynomial P is positive when $P(x) > 0$ for $x \in I$. However, in some cases it is assumed that B_0 vanishes at some points of I.

the new weight function. In the second part of the talk we show how can be applied a new formulation found in [3], to calculate the quadrature coefficients for the modified weight B_0W/A_0.

This talk is based on a work carried out jointly with F. Cala Rodríguez (Universidad Austral de Chile) and the research we are currently conducting at University of Vigo with A. Cachafeiro, E. Berriochoa and E. Martínez.

Keywords: Gauss quadrature formula, interpolatory rule, rational quadrature formula, rate of convergence.

AMS Classification: 41A55, 65D32.

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Some remarks on the iterates of positive linear operators*

Mircea Ivan

Abstract

The outstanding results of Kelisky and Rivlin [1] and Karlin and Ziegler [2] provided new insights into the study of the limit behavior of the iterates of linear operators defined on $C[0,1]$. They have attracted a lot of attention lately and several new proofs and generalizations have been given in the last fifty years (see the references). Nevertheless, the general problem concerning the over-iterates of positive linear operators remained unsolved.

In this note we present a general result concerning the limit of the iterates of positive linear operators. As applications, we deduce the asymptotic behaviour of the iterates of almost all classic and new positive linear operators.

A preliminary version of this work was communicated during the Approximation & Computation 2008 conference, August 25-29, Niš, Serbia.

Keywords: Positive linear operators, iterates, Bernstein operators, Meyer-König and Zeller operators.

AMS Classification: 46A32, 41A36.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Hölder spaces of sign sensitive weighted integrable functions

Miguel A. Jiménez-Pozo and José M. Hernández-Morales

Abstract

Let $L^1(w)$ be the asymmetric normed space of $2\pi-$periodic real-valued functions which are integrable with respect to a sign sensitive weight $w = (w_+, w_-)$. We introduce in a natural way a modulus of smoothness for functions in $L^1(w)$ and then the associated spaces $\mathrm{Lip}^1_\alpha(w)$ and $\mathrm{lip}^1_\alpha(w)$ of Hölder (or Lipschitz) functions. In this context, trigonometric polynomial approximation and other topological properties of these spaces are studied.

Keywords: Hölder approximation, Lipschitz functions, sign-sensitive weights, weighted integrals, 0-equicontinuous sets.

AMS Classification: (primary) 41A65; (secondary) 41A36, 41A25, 42A10.

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• I Jaen Conference on Approximation Theory
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Zeros of polynomials embedded in an orthogonal sequence

Alan Beardon, Kathy Driver and Kerstin Jordaan

Abstract

Let $\{x_{i,n}\}_{i=1}^{n}$ and $\{x_{i,n+1}\}_{i=1}^{n+1}$, $n \in \mathbb{N}$, be two given sets of real distinct points with $x_{1,n+1} < x_{1,n} < x_{2,n+1} < \cdots < x_{n,n} < x_{n+1,n+1}$. Wendroff (cf. [1]) proved that if $p_n(x) = \prod_{i=1}^{n}(x - x_{i,n})$ and $p_{n+1}(x) = \prod_{i=1}^{n+1}(x - x_{i,n+1})$ then p_n and p_{n+1} can be embedded in a non-unique infinite monic orthogonal sequence $\{p_n\}_{n=0}^{\infty}$. We investigate the connection between the zeros of p_{n+2} and the two coefficients $b_{n+1} \in \mathbb{R}$ and $\lambda_{n+1} > 0$ that define p_{n+2} via the three term recurrence relation

$$p_{n+2}(x) = (x - b_{n+1})p_{n+1}(x) - \lambda_{n+1}p_n(x).$$

We show how the relative positions of the zeros of p_n, p_{n+1} and p_{n+2} depend on b_{n+1} and we derive bounds for λ_{n+1}.

Keywords: Interlacing zeros, construction of orthogonal sequences, three term recurrence relation, Wendroff's Theorem.

AMS Classification: 33C45, 42C05.

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On approximations by some non-standard trigonometric systems*

Andi Kivinukk

Abstract

A standard approximation tool for a 2π-periodic function is its Fourier series using the system

$$\{1,\ \cos kx,\ \sin kx\}_{k=1}^{\infty}, \tag{1}$$

which is orthogonal on $[-\pi,\pi]$ and closed in $C_{2\pi}$. The elements of that system are 2π-periodic, i. e.

$$f(\pi+x)=f(x-\pi), \quad x\in\mathbb{R}. \tag{A}$$

Three others, quite natural orthogonal on $[-\pi,\pi]$ systems could be interest of a study. These are

$$\{\cos(k-1/2)x,\ \sin(k-1/2)x\}_{k=1}^{\infty}, \tag{2}$$
$$\{1,\ \cos kx,\ \sin(k-1/2)x\}_{k=1}^{\infty}, \tag{3}$$
$$\{\cos(k-1/2)x,\ \sin kx\}_{k=1}^{\infty}, \tag{4}$$

which are not mentioned in literature in a systematic way, cf. [1, 2]. These systems satisfy conditions, respectively,

- $f(\pi+x)=-f(x-\pi), \quad x\in\mathbb{R}$ (B)
  (f is said to be 2π-antiperiodic);
- $f(\pi+x)=f(\pi-x), \quad x\in\mathbb{R}$ (C)
  (f is said to be π-symmetric);
- $f(\pi+x)=-f(\pi-\mathrm{x}), \quad x\in\mathbb{R}$ (D)
  (f is said to be π-antisymmetric);

Conditions (A) and (B) yield 4π-periodicity, but this is not true for (C) and (D), see examples $f(x)=(\pi-x)^k$ with $k=1;2$.

*This is joint work with O. Labanova and M. Zeltser (Tallinn University). The research was supported in part by the Estonian Science Foundation grant 7033.

We will study the system (3). We denote by $\overline{C}_{4\pi}$ the set of all π-symmetric and 4π-periodic functions. Since the system (3) is orthogonal on $[-\pi,\pi]$ and closed in $f\in\overline{C}_{4\pi}$ we may associate its π-symmetric partial Fourier sums

$$S_n^C(f,x)=\frac{a_0}{2}+\sum_{k=1}^{n}a_k\cos kx+d_k\sin\left(k-1/2\right)\,x$$

with the real Fourier coefficients

$$a_k:=\frac{1}{\pi}\int_{-\pi}^{\pi}f(t)\cos ktdt,\quad d_k:=\frac{1}{\pi}\int_{-\pi}^{\pi}f(t)\sin(k-1/2)tdt.$$

It turns out that for some functions π-symmetric Fourier series converges faster than the classical trigonometric Fourier series. The corresponding polynomial approximation operator is

$$U_n\left(f,x\right):=$$
$$\frac{a_0}{2}+\sum_{k=1}^{n}\left(\varphi\left(\frac{k}{n+1}\right)\ a_k\cos kx+\varphi\left(\frac{k-1/2}{n+1}\right)d_k\sin\left(k-\frac{1}{2}\right)x\right),$$

where $\varphi\in C_{[0,1]}$, $\varphi\left(0\right)=1,\ \varphi\left(1\right)=0$.

The typical result for the order of approximation is as follows.

Theorem 1. *Let $f\in\overline{C}_{4\pi}$ and let U_n be ultrapositive operator, then*

$$\|\,U_nf-f\,\|_{\overline{C}_{4\pi}}\le 4\omega_2\left(f,\left(1-\varphi\left(\frac{1}{n+1}\right)\right)^{1/2}\right)+\|\,f\,\|_{\overline{C}_{4\pi}}\left(1-l_n\right),$$

where ω_2 is the modulus of smoothness and

$$l_n=\frac{4}{\pi}\sum_{k=1}^{n}\frac{\left(-1\right)^{k-1}}{2k-1}\varphi\left(\frac{k-1/2}{n+1}\right)$$

Remark 2. If $\varphi\in Lip\ 1$, then $\lim_{n\to\infty}l_n=1$.

Keywords: Non-standard trigonometric system, order of approximation.
AMS Classification: 41A25, 42A10.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Positive trigonometric sums and applications to starlike functions*

Stamatis Koumandos

Abstract

Let $d_k = \frac{(\mu)_k}{k!} = \frac{\Gamma(k+\mu)}{\Gamma(\mu)\,k!}$, $k = 0, 1, 2\ldots$, $\mu \in (0,\, 1]$ and let $s_n^\mu(z) := \sum_{k=0}^n d_k\, z^k$ be the nth partial sum of the Taylor expansion of $(1-z)^{-\mu}$ around the origin. For all $\rho \in (0, 1]$ and $n \in \mathbb{N}$ we show that

$$|\arg[(1-z)^\rho s_n^\mu(z)]| \leq \rho\pi/2,\ |z| < 1$$

is equivalent to the trigonometric inequality

$$\sum_{k=0}^n d_k \sin(2k+\rho)\theta > 0,$$

for all $n = 1, 2, \ldots$ and $0 < \theta < \pi$. It is conjectured that this inequality holds precisely when $0 < \mu \leq \mu^*(\rho)$, where $\mu^*(\rho)$ is the unique solution $\mu \in (0, 1]$ of

$$\int_0^{(\rho+1)\pi} \sin\big(t - \rho\pi\big) t^{\mu-1}\, dt = 0\,.$$

We present several special cases of this conjecture together with their interpretation in geometric function theory. Some properties and approximations of the function $\mu^*(\rho)$ are also given.

This work is closely related to recent investigations concerning generalizations and extensions of the celebrated Vietoris' inequalities for trigonometric sums. Far-reaching extensions and results on starlike functions of order $1 - \mu/2$ are discussed.

Our proofs rely on several sharp inequalities for special functions. Many of these inequalities are obtained by verifying the complete monotonicity of certain functions via Bernstein's famous theorem. In particular, we find the best possible lower bound μ_0 such that the derivative of $x - \frac{\Gamma(x+\mu)}{\Gamma(x+1)} x^{2-\mu}$ is completely monotonic on $(0, \infty)$ for $\mu_0 \leq \mu < 1$.

Keywords: Positive trigonometric sums, sharp inequalities, approximation, starlike functions, subordination, convolution, Gamma function, completely monotonic functions.

AMS Classification: 42A05, 30C45, 26D05, 26D15, 33C45, 33B15, 41A58.

*This research was supported by the grant number 3411-21041 from the Leventis Foundation

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• I Jaen Conference on Approximation Theory
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On the approximation of regular convex bodies by convex algebraic level surfaces*

András Kroó

Abstract

We consider in this talk the problem of the approximation of regular convex bodies in $\mathbb{R}^d$ by level surfaces of **convex** algebraic polynomials. P.C. Hammer verified that any convex body in $\mathbb{R}^d$ can be approximated by a level surface of a convex algebraic polynomial. Subsequently we gave a quantitative version of Hammer's approximation theorem by showing that the order of approximation of convex bodies by convex algebraic level surfaces of degree n is $\frac{1}{n}$. Moreover, it was also verified that whenever the convex body is not regular (that is there exists a point on its boundary at which the convex body possesses two distinct supporting hyper planes) then $\frac{1}{n}$ is essentially the sharp rate of approximation. This leads to the natural question wether this rate of approximation can be improved further when the convex body is **regular**. It turns out that for any **regular convex body** which possesses a unique supporting hyper plane at each point of its boundary a $o(1/n)$ rate of convergence holds. In addition, if the body satisfies the condition of C^2**-smoothness** the rate of approximation is $O(\frac{1}{n^2})$.

Keywords: Convex level surfaces of algebraic polynomials, convex bodies, rate of approximation.

AMS Classification: 41A17.

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*Supported by the OTKA Grant T77812.

On a modification of Szász-Mirakjan-Kantorovich operators

Francesco Altomare, Mirella Cappelletti Montano and Vita Leonessa

Abstract

The Szász-Mirakjan-Kantorovich operators were first introduced by P.L. Butzer (see [2]) in order to obtain an approximation process for spaces of integrable functions on unbounded intervals.
We recall that they are defined by setting

$$K_n(f)(x) := \sum_{k=0}^{\infty} e^{-nx} \frac{(nx)^k}{k!} \left[n \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)\, dt \right],$$

for every $f \in L^1([0, +\infty[)$, $x \geq 0$ and $n \geq 1$.

In this talk we present some recent results concerning a further modification of such operators.

Our idea follows from that one developed in [1] where the authors dealt with a generalization of the Kantorovich operators. Namely, if $(a_n)_{n\geq1}$ and $(b_n)_{n\geq1}$ are two sequences of real numbers satisfying $0 \leq a_n < b_n \leq 1$ for every $n \geq 1$, then our operators are defined as follows

$$C_n(f)(x) := \sum_{k=0}^{\infty} e^{-nx} \frac{(nx)^k}{k!} \left[\frac{n}{b_n - a_n} \int_{\frac{k+a_n}{n}}^{\frac{k+b_n}{n}} f(t)\, dt \right],$$

for every $f \in L^1([0, +\infty[)$, $x \geq 0$ and $n \geq 1$.

A possible interest in the study of C_n's rests on the fact that, by means of them, it is possible to reconstruct a continuous or an integrable function on $[0, +\infty[$ by knowing the mean values of it on a finite number of subintervals of $[0, +\infty[$ which does not necessarily constitute a subdivision.

We investigate their approximation properties both on several continuous function spaces and on Lebesgue spaces. We also collect several estimates of the rate of convergence by means of suitable moduli of smoothness.

The results which will be discussed during the talk are taken from [3].

Keywords: Szász-Mirakjan-Kantorovich operator, positive approximation process, weighted space, Korovkin-type approximation theorem, modulus of smoothness.

AMS Classification: 41A10, 41A25, 41A36.

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The use of multi-point Taylor expansions in one-dimensional boundary value problems*

José L. López, Ester Pérez Sinusía and N. M. Temme

Abstract

We consider second-order linear differential equations $\varphi(x)y'' + f(x)y' + g(x)y = h(x)$ in the interval $(-1, 1)$ with Dirichlet, Neumann or mixed Dirichlet-Neumann boundary conditions given at the extreme points. We consider $\varphi(x)$, $f(x)$, $g(x)$ and $h(x)$ analytic in a Cassini disk with foci at $x = \pm 1$ containing the interval $(-1, 1)$. The two-point Taylor expansion of the solution $y(x)$ at the extreme points ± 1 is used to give a criterion for the existence and uniqueness of solution of the boundary value problem. This method is constructive and provides the two-point Taylor approximation of the solution(s) when it exists.

We also consider the same differential equation but with Dirichlet, Neumann or mixed Dirichlet-Neumann boundary conditions given at three points of the interval: the two extreme points $x = \pm 1$ and an interior point $x = s \in (-1, 1)$. In this case, a three-point Taylor expansion of the solution $y(x)$ at the extreme points ± 1 and at $x = s$ is used to give a criterion for the existence and uniqueness of the solution of the boundary value problem.

Keywords: Second-order linear differential equations, boundary value problems, frobenius method, multi-point Taylor expansions.

AMS Classification: 35A35, 41A58.

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*The *Ministerio de Ciencia y Tecnología* (REF. MTM2007-63772) and the *Gobierno de Navarra* (Res. 228/2008) are acknowledged by their financial support.

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Spline wavelet decomposition for irregular grid on segment *

Yuri K. Demjanovich and Anton A. Makarov

Abstract

Wavelet decompositions were constructed in many works. In many papers refinement equations (uniform grids) or lifting schemes (irregular grids are allowed) were taken for initial relations. In the case of spline wavelet decomposition for irregular grid the role of initial relations was played by approximation relations combined with a special projection operator (cf. [1–3]). Our goal is to construct a wavelet decomposition and the corresponding decomposition/reconstruction algorithms in the case of finite flow (with an irregular grid on a segment) for a space of minimal splines.

The source of minimal splines is approximation relations regarded as a system of linear algebraic equations. Normalized splines (called B_φ-splines) of the third order on a closed interval are constructed. The finite-dimensional space of splines (nonpolynomial in general) of the class C^2 are constructed. The splines have the minimal compact support. A realization of system of biorthogonal functionals for B_φ-splines is proposed. The solutions of some interpolation problems generated by mentioned biorthogonal system are offered.

An embedding of the general spline spaces is established for arbitrary refinements of irregular grid. This leads to a wavelet decomposition (e. g. signals with fast oscillations). Spline wavelet decomposition in the case of sequence of refining irregular grids is discussed. The basis wavelets are compactly supported and admit simple analytic representation. The formulas of decomposition and reconstruction are done. Obtained formulas are easily parallelized.

Keywords: Approximation, interpolation, splines, wavelets.
AMS Classification: 41A05, 41A15, 42C40, 65T60.

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*The work was partially supported by RFBR (grant 10-01-00245).

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Asymptotics of the L^2 norms of derivatives of OPUC

Andrei Martínez–Finkelshtein and Barry Simon

Abstract

Let μ be a non-trivial Borel measure on the unit circle $\mathbb{T}$, and $\{\varphi_n\}$ the corresponding sequence of orthonormal polynomials (OPUC). Let us denote by

$$\|f\|_{L^2_\mu} = \left(\int |f|^2 \, d\mu \right)^{1/2}$$

the weighted L^2 norm. In this talk we will focus on one question from the asymptotic theory of OPUC. Namely, we say that μ has normal L^2–derivative behavior (is *normal*, for short), if

$$\lim_n \left\| \frac{\varphi_n'}{n} \right\|_{L^2_\mu} = 1 \,. \tag{1}$$

We will reveal some connections of this notion with different relevant objects in the theory of OPUC, and present some necessary/sufficient conditions for (1).

Keywords: asymptotics, orthogonal polynomials on the unit circle, Nevai condition, Szegő condition.

AMS Classification: 42C05.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

A sufficient condition for Marcikiewicz-Zygmund inequalities in terms of the mesh norm*

Jordi Marzo

Abstract

We relate arrays of points on the sphere for which the Marcinkiewicz-Zygmund inequality holds, with the uniqueness sets for a certain space of entire functions. By a uniqueness criterion due to A. Beurling, we obtain a sufficient condition for the Marcinkiewicz-Zygmund inequality to hold. This is expressed in terms of the mesh norm of the arrays. We apply the condition obtained to different arrays of points on the sphere.

Keywords: Spherical harmonics, Marcinkiewicz-Zygmund inequalities, mesh norm.

AMS Classification: 65D32, 33C55, 65T40, 11K36.

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*Research Council of Norway grant 185359/V30

Polynomial approximation with exponential weights in $(-1,1)$

G. Mastroianni and I. Notarangelo

Abstract

In this talk I will discuss the polynomial approximation in $(-1,1)$ with the weight

$$w(x) = \mathrm{e}^{-(1-x^2)^{-\alpha}}, \qquad \alpha > 0 .$$

I am going to complete some results already treated last year in the X International Meeting on Approximation Theory of the University of Jaén.

I will also consider the approximation by means of Fourier sums and Lagrange interpolation.

Keywords: Jackson theorems, approximation by polynomials, orthogonal polynomials, Fourier series, Lagrange interpolation.

AMS Classification: 41A10, 42C10, 42A20, 41A05.

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Smoothing the Gibbs phenomenon using Padé-Hermite approximants

Ana Matos, Bernd Beckermann, Valery Kaliaguine and Franck Wielonsky

Abstract

In order to reduce the Gibbs phenomenon exhibited by the partial Fourier sums of a periodic function f, defined on $[-\pi, \pi]$, discontinuous at 0, Driscoll and Fornberg [3] suggested the construction of a class of approximants which incorporate the knowledge of that singularity. More precisely, their approach is the following one: let g_2 denote the series such that $f(t) = \Re(g_2(e^{it}))$. Then, the goal is to approach g_2 on the unit circle (and more precisely its real part). It is typical that the singularity of the function f, located at 0 say, corresponds to a logarithmic singularity for g_2, then located at 1, and that this function g_2 is analytic in the complex plane, with a branch cut that can be taken as the interval $[1, \infty)$. Defining $g_1(z) = \log(1 - z)$, we may consider the problem of determining polynomials p_0, p_1, p_2 such that

$$p_0(z) + p_1(z)g_1(z) + p_2(z)g_2(z) = O(z^{n_0+n_1+n_2+2}) \quad (z \to 0)$$

where n_j denotes the degree of p_j, $j = 0, 1, 2$. We can then propose the *Hermite-Padé approximant*

$$\Pi_{\vec{n}}(z) = -\frac{p_0(z) + p_1(z)g_1(z)}{p_2(z)},$$

to approximate g_2. Note that when $p_1(z) = 0$
(or formally $n_1 = -1$) we recover the usual Padé approximant of g_2 of type (n_0, n_2).

Convincing numerical experiments have been obtained by Driscoll and Fornberg, but no error estimates have been proven so far.

In this talk we obtain rates of convergence of sequences of Hermite–Padé approximants for a class of functions known as *Nikishin systems*. Our theoretical findings and numerical experiments confirm that particular sequences of Hermite-Padé approximants (diagonal and row sequences, as well as linear HP approximants) are more efficient than the more elementary Padé approximants, particularly around the discontinuity of the goal function f.

Keywords: Gibbs penhomenon, Hermite-Padé approximants, logarithmic potential theory, orthogonal polynomials.

AMS Classification: 41A21, 41A20, 41A28, 42A16, 31C15, 31C20.

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• Úbeda, Jaén, Spain, July 4th-9th, 2010

Extended Chebyshev spaces and weight functions

Marie-Laurence Mazure

Abstract

Extended Chebyshev spaces are classical tools in Approximation Theory. More recently they also have been intensively used for geometric design purposes because of the powerful shape parameters they provide.

On a given closed bounded interval it is known that a given Extended Chebyshev space can be written as the kernel of a linear differential operator associated with a system of positive weight functions. We determine all systems of weight functions which can be used. This has important applications, e.g., it enables us to build all possible Chebyshevian splines with prescribed section spaces which are "good" for design or interpolation. The result is also closely connected with the construction of Bernstein-Chebyshev operators.

Keywords: Extended Chebyshev spaces, weight functions, generalized derivatives, Bernstein-type bases, blossoms.

AMS Classification: 41A50, 41A30, 65D05, 65D17.

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• I Jaen Conference on Approximation Theory
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Universal meromorphic approximation

Thierry Meyrath

Abstract

The first example of a so-called universal function is due to Birkhoff, who constructed in 1929 an entire function f with the property, that the sequence $\{f(z+n) : n \in \mathbb{N}\}$ is dense in the space of all entire functions. Since then, a large number of results concerning universality have been discovered, mainly dealing with holomorphic and entire functions. We shall investigate the problem of universal meromorphic approximation, showing the existence of meromorphic functions with universal properties and extending some of the classic results to the meromorphic case.

Keywords: Universal functions, meromorphic approximation.
AMS Classification: 30E10, 30K99.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Multiplicative perturbations of the Laplacian and related approximation problems

Francesco Altomare, Sabina Milella and Graziana Musceo

Abstract

In this talk we present some recent results concerning the operator

$$\alpha\Delta u(x) = \sum_{i=1}^{n} \alpha(x) \frac{\partial^2 u}{\partial x_i^2}(x) \quad (x \in \mathbb{R}^n).$$

We prove that, if $\alpha\Delta$ is defined on the maximal domain

$$D(\alpha\Delta) = \left\{ u \in C_0^w(\mathbb{R}^n) \cap C^2(\mathbb{R}^n) \mid \alpha\Delta u \in C_0^w(\mathbb{R}^n) \right\},$$

then it is closable and its closure generates a positive quasi contractive C_0-semigroup $(T(t))_{t\geq 0}$ which fulfils the Feller property (i.e., it leaves invariant $C_0(\mathbb{R}^n)$ and it is a contractive semigroup on it) and which is associated with a suitable probability transition function on $\mathbb{R}^n$ and hence with a Markov process on $\mathbb{R}^n$.

Here $\alpha, w \in C_b(\mathbb{R}^n)$ are strictly positive functions and $C_0^w(\mathbb{R}^n)$ denotes the weighted space of function $f \in C(\mathbb{R}^n)$ such that $\lim\limits_{\|x\|\to+\infty} w(x)f(x) = 0$.

In the case of polynomial weights, we state that the semigroup $(T(t))_{t\geq 0}$ can be approximated by means of iterates of the integral operators

$$G_n(f)(x) := \frac{1}{(2\pi)^{\frac{N}{2}}} \int_{\mathbb{R}^N} f\left(\sqrt{\frac{2\alpha(x)}{n}}y + x\right) e^{-\frac{\|y\|^2}{2}} dy$$

($f \in C_0^w(\mathbb{R}^n)$, $x \in \mathbb{R}^n$, $n \geq 1$).

Such an approximation formula allow us to disclose some properties of $(T(t))_{t\geq 0}$ and of the random variables which govern the relevant Markov process.

Keywords: Second order elliptic differential operator, positive C_0-semigroup, positive approximation process, integral operator, asymptotic formula, shape preserving approximation.

AMS Classification: 47D06, 41A10, 41A36, 35A35.

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On the construction of H-Bases

H. Michael Möller and Tomas Sauer

Abstract

The H-basis concept allows, similar to the Gröbner basis concept, a reformulation of nonlinear problems in terms of linear algebra, esp. the polynomial interpolation problem and the solving of systems of polynomial equations, [2]. H-bases are typically constructed by using Buchberger's algorithm, i.e., by constructing first a Gröbner basis. Only in exceptional cases [1] a direct method is known. Here we present an algorithm which computes directly an H-basis, whenever a finite set of polynomials is given which generate the same ideal.

Keywords: Multivariate polynomials, Gröbner bases, interpolation, systems of polynomial equations.

AMS Classification: 65D05, 65H10, 13P10.

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All the families in the Askey-scheme are orthogonal in the whole range of their parameters*

Samuel G. Moreno and Esther M. García–Caballero

Abstract

The parameters for each family of hypergeometric polynomials in the Askey-scheme are classically restricted in order to ensure either the orthogonality with respect to a positive-definite moment functional, or a finite orthogonality relation. All these families can be easily extended in such a way that wider sets of parameters are allowed. For some of the new considered parameters, Favard's theorem ensures that the extended families still satisfy an orthogonality condition, now with respect to a quasi-definite moment functio-nal. For the other ones, for which the three term recurrence relation breaks down, no orthogonality condition can be ensured. The aim of this talk is to show that each of the hypergeometric families in the Askey-scheme still fulfills an orthogonality condition even in those intriguing cases in which the hypothesis of Favard's theorem does not success.

We also conjecture that our approach can be applied to establish the orthogonality for all the families in the q-Askey scheme in the whole range of their parameters.

Keywords: hypergeometric orthogonal polynomials, Favard's theorem, non-standard orthogonality.

AMS Classification: 33C45, 42C05.

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*This work was partially supported by Junta de Andalucía, Research Group FQM 0178, and by Ministerio de Educación y Ciencia, Project MTM2006-14590.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

On Fourier-Jacobi series convergence in average

Vitalii P. Motornyi, Oxana V. Motorna and Sergiy V. Goncharov

Abstract

We investigate the Fourier-Jacobi series convergence in the cases, when Lebesgue constants are unbounded.

Let $P_n^{\alpha,\beta}(x)$ be Jacobi polynomial, orthogonal on $[-1,1]$ with the weight $\rho(x) = (1-x)^\alpha(1+x)^\beta$, $\alpha > -1, \beta > -1$. Denote by $L_{p,A,B}$ the space of measurable on $[-1,1]$ functions f, such that $f\omega^{1/p} \in L^p$, where the weighted function $\omega(x) = (1-x)^A(1+x)^B$, $A > -1, B > -1$. The norm $\|f\|_{p,A,B} = \|f\omega^{1/p}\|_p$. If $A = B = 1$, then $L_{p,1,1} = L_p$ and $\|f\|_{p,1,1} = \|f\|_p = (\int_{-1}^1 |f(x)|dx)^{1/p}$. Denote by $S_n^{\alpha,\beta}(f)$ the n-th partial sum of the Fourier-Jacobi series of the function $f \in L_{1,\alpha,\beta}$. Finally, let $H^{r,\gamma}$, $r \in \mathbb{N}, 0 < \gamma \leq 1$, be the set of functions f, that have the r-th derivative $f^{(r)} \in L_p$, satisfying

$$\left(\int_{-1}^{1-h} |f^{(r)}(x) - f^{(r)}(x+h)|^p dx\right)^{1/p} \leq Ch^\gamma, \quad 0 < h < 1.$$

So, we have

Theorem 1. *Let* $1 < p < \infty$, $\alpha = \beta$; $\mu = \nu = (2A+2)/p - \alpha - 3/2 \geq 0$, $A = B \in (-1/2; 0)$, $f \in H_p^{r+\gamma}$. *Then*

$$\|f - S_n^{(\alpha,\beta)}(f)\|_{p,A,A} \leq \begin{cases} \frac{C_\gamma}{n^{r+\gamma}}, & \text{if} \quad \gamma > \mu - \frac{2A}{p}, \\ C\frac{\ln^{1/p}(n+1)}{n^{-\mu+2A/p+2\gamma}}, & \text{if} \quad \frac{\mu}{2} - \frac{A}{p} < \gamma \leq \mu - \frac{2A}{p}. \end{cases}$$

Keywords: Fourier-Jacobi series, convergence, Lebesgue constants.
AMS Classification: 41A25.

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• I Jaen Conference on Approximation Theory
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Lacunary Runge type approximation by means of the Hadamard product

J. Müller and T. Pohlen

Abstract

Let K be a compact subset of the complex plane with connected complement, and let $(C(K), \|\cdot\|)$ be the Banach space of functions continuous on K with the uniform norm. For a subset Λ of the nonnegative integers $\mathbb{N}_0$ we denote by $P_\Lambda(K)$ the closed linear span of the monomials $z \mapsto z^n$ $(n \in \Lambda)$ in $C(K)$. Runge's classical theorem on polynomial approximation says that every function on K which extends holomorphically to a neighborhood of K belongs to $P_{\mathbb{N}_0}(K)$. Let $H(K)$ be the subspace of $C(K)$ of such functions. It is known that in the case $0 \notin K$ "lacunary versions" of Runge's theorem hold, that is, $H(K) \subset P_\Lambda(K)$ for certain $\Lambda \subset \mathbb{N}_0$.

We present a new approach to such kind of results which is based on the Hadamard product $\sum_{\nu=0}^{\infty} a_\nu b_\nu z^\nu$ of two power series $\sum_{\nu=0}^{\infty} a_\nu z^\nu$ and $\sum_{\nu=0}^{\infty} b_\nu z^\nu$.

Keywords: Hadamard product, lacunary approximation, gap series.

AMS Classification: 30E10.

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Second order differential operators and modified Szász-Mirakjan operators

Francesco Altomare, Sabina Milella and Graziana Musceo

Abstract

In this talk we present some generation results of strongly continuous positive semigroups for second order differential operators of the form

$$Lu(x) = xu''(x) + \beta(x)u'(x) + \gamma(x)u(x) \quad (x > 0) \tag{1}$$

in the framework of weighted continuous function spaces.
More precisely, we provide some conditions on the coefficients β and γ which guarantee generation property for the operator (1) equipped with Wentzell-type boundary conditions

$$\lim_{x\to 0^+} xu''(x) + \beta(x)u'(x) = 0 \ \text{ and } \ \lim_{x\to+\infty} \frac{xu''(x) + \beta(x)}{1 + x^m} = 0 \tag{2}$$

and with maximal-Wentzell type boundary conditions

$$\lim_{x\to 0^+} xu''(x) + \beta(x)u'(x) \in \mathbb{R} \ \text{ and } \ \lim_{x\to+\infty} \frac{xu''(x) + \beta(x)}{1 + x^m} = 0 \tag{3}$$

in the setting of the weighted spaces

$$E_m^* := \{u \in C(]0, +\infty[) : \lim_{\substack{x\to 0^+\\ x\to+\infty}} \frac{u(x)}{1 + x^m} \in \mathbb{R}\}$$

and

$$E_m^0 := \{u \in C([0, +\infty[) : \lim_{x\to+\infty} \frac{u(x)}{1 + x^m} = 0\}$$

$(m \geq 2)$.
Moreover, in the setting of E_m^0 we search a suitable sequence of positive linear operators

whose iterates approximate the semigroup $(T(t))_{t\geq 0}$ above, and then the solutions to the initial-boundary value problem associated with the operator L

$$\begin{cases} \dfrac{\partial u}{\partial t} = Lu(x,t), \\ \\ u(x,0) = u_0(x), \\ u(\cdot,t) \text{ verifies (2) or (3)} \quad \text{(for every } t \geq 0). \end{cases} \tag{4}$$

To this purpose we construct a suitable sequence of operators by modifying the classical Szász-Mirakjan operators, as follows

$$S_n^*(f)(x) := \sum_{k=0}^{+\infty} e^{-nx} \frac{(nx)^k}{k!} \left(1 + \frac{\gamma(k/n)}{2n}\right) f\left(\frac{k}{n} + \frac{\beta(k/n)}{2n}\right)$$

and we show that

$$T(t) = \lim_{n\to\infty} (S_n^*)^{k(n)} \quad \text{strongly on } E_m^0$$

for every $t \geq 0$ and for every sequence $(k(n))_{n\geq 1}$ of positive integers such that $\dfrac{k(n)}{n} \to t$. Such a representation formula allows us to derive some qualitative information on the semigroup and hence on the solutions to (4).

Keywords: Degenerate second order differential operators, positive C_0-semigroups, positive approximation processes, modified Szász-Mirakjan operators.

AMS Classification: 47D06, 41A10, 41A36.

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• I Jaen Conference on Approximation Theory
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Asymptotics for ménage Polynomials

Thorsten Neuschel

Abstract

These polynomials are defined through

$$U_n(t) = \sum_{k=0}^{n} (t-1)^k \frac{2n}{2n-k} \binom{2n-k}{k} (n-k)!$$

arising in combinatorics where occasionally they are termed as menage hit polynomials. The special values $U_n(0)$ are the well known reduced menage numbers. Using a connection formula involving certain ${}_3F_1$-polynomials strong and weak asymptotics for suitably normalized menage polynomials are derived. The proofs rely on the asymptotic evaluation of parameter integrals combined with arguments from potential theory.

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Polynomial inequalities and embedding theorems with exponential weights on $(-1, 1)$

I. Notarangelo

Abstract

For $x \in (-1, 1)$ we consider a weight function of the form $u = vw$, where

$$w(x) = \mathrm{e}^{-(1-x^2)^{-\alpha}}, \qquad \alpha > 0,$$

and v is a doubling weight. Under suitable assumptions on the weight u, we prove some polynomial inequalities, e.g. Remez-, Bernstein–Markoff-, Nikolskii- and Schur-type inequalities in weighted L^p-norm, $1 \leq p \leq \infty$. As an application, we state some embedding theorems, extending some results due to Ul'yanov.

Keywords: Weighted polynomial inequalities, exponential weights, doubling weights, embedding theorems.

AMS Classification: 41A17.

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Positive rational interpolatory quadrature rules on the interval and the complex unit circle with preassigned nodes*

K. Deckers, A. Bultheel and F. Perdomo-Pío

Abstract

We consider rational Gauss-type quadrature rules on the interval $[-1,1]$ with one fixed node in $x_\alpha = \cos(\theta_\alpha) \in (-1,1)$. The remaining nodes are then chosen inside the interval $[-1,1]$ to achieve the maximal possible domain of validity in the space of rational functions, while maintaining positive weights. In this contribution we discuss the connection of these quadrature rules with rational Szegő-Lobatto quadrature rules on the complex unit circle with fixed nodes in $z_\alpha = e^{i\theta_\alpha}$ and $z_\beta = \overline{z}_\alpha$.

Keywords: Rational Gauss-type quadratures, rational Szegő-Lobatto quadratures.
AMS Classification: 41A55, 42C05, 65D32.

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*The first author is a Postdoctoral Fellow of the Research Foundation - Flanders (FWO).
The work of the first two authors is partially supported by the Belgian Network DYSCO (Dynamical Systems, Control, and Optimization), funded by the Interuniversity Attraction Poles Programme, initiated by the Belgian State, Science Policy Office. The scientific responsibility rests with its authors.
The work of the third author has been partially supported by a Grant of Agencia Canaria de Investigación, Innovación y Sociedad de la Información del Gobierno de Canarias.

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Best uniform polynomial approximation to $1/x^*$

Johannes Kraus, Veronika Pillwein and Ludmil Zikatanov

Abstract

Many years ago, dealing with special functions was a tedious, time-consuming, and error-prone task, which required long training, skillful manipulations and structural insight. This is now mostly obsolete: today there is an increasing number of symbolic algorithms available which are capable of dealing with special functions [1].

In particular, questions about orthogonal polynomials which often arise in numerical mathematics can be answered by packages for holonomic functions. With these programs, dealing with special functions is straight-forward, fast, and reliable.

Recently we used an algorithm developed by Manuel Kauers [3] for deriving a three term recurrence for the polynomial of best uniform approximation to $1/x$ on a finite interval [2, 4]. This recurrence relation entered the analysis of an algebraic multilevel iteration method. We will give an introduction to the underlying symbolic algorithm and its scope, and sketch our application to algebraic multigrid methods.

Keywords: Special functions, symbolic computation, numerical analysis.

AMS Classification: 33F10, 33C45, 65N55, 41A50.

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*the second named author was supported by the Austrian Science Fund (FWF) under grant W1214/DK6

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Existence of monotonically complemented subspaces and related results

Michael Prophet and Douglas Mupasiri

Abstract

With $X = (C[a,b], \|\cdot\|)_\infty$, we consider the existence of subspaces $V \subset X$ which are *monotonically complemented*; that is, V such that there exists a projection $P : X \to V$ preserving monotonicity. In this talk we will show that there are 'relatively few' such spaces; for example, any Haar subspace of dimension three (or greater) which contains the constant function fails to be monotonically complemented. We will also indicate how these results to can be extended to the preservation of other characteristics or shapes.

Keywords: Projections, shape preserving approximation.

AMS Classification: 41A28, 41A40, 41A60.

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On the semiclassical character of orthogonal polynomials satisfying structure relations*

Amílcar Branquinho and Maria das Neves Rebocho

Abstract

In this talk we prove the semiclassical character of some sequences of orthogonal polynomials, say $\{P_n\}$, $\{R_n\}$, related through relations of the following type: $\sum_{k=0}^{N} \zeta_{n,k} R^{(\alpha)}_{n+i-k} = \sum_{k=0}^{M} \xi_{n,k} P_{n+j-k}$, where i, j, M, N, α are non-negative integers, $\zeta_{n,k}, \xi_{n,k}$ are complex numbers, and $R^{(\alpha)}$ denotes the α-derivative of R. The case $M = j = 0, \alpha = 2, i = 2$ is studied for a pair of orthogonal polynomials whose corresponding orthogonality measures are coherent. The relation $\sum_{k=0}^{s} \xi_{n,k} P_{n+s-k} = \sum_{k=0}^{s+2} \zeta_{n,k} P'_{n+s+1-k}$ is shown to give a characterization for the semiclassical character of $\{P_n\}$.

Keywords: Orthogonal polynomials, recurrence relations, semiclassical linear functionals, structure relations.

AMS Classification: 33C45, 42C05.

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*The work of Maria das Neves Rebocho was supported by FCT, under grant SFRH/BPD/45321/2008.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Asymptotic behavior for Sobolev orthogonal polynomials with non-diagonal Sobolev products*

Ana Portilla, Yamilet Quintana, José M. Rodríguez
and Eva Tourís

Abstract

Sobolev orthogonal polynomials have been more and more investigated in recent years. One of the central problems in the theory of Sobolev orthogonal polynomials is to determine their asymptotic behavior. In [3] the authors show how to obtain the n-th root asymptotics of Sobolev orthogonal polynomials if the zeros of these polynomials are contained in a compact set of the complex plane. Although the uniform bound of the zeros of orthogonal polynomials holds for every measure with compact support in the case without derivatives, it is an open problem to bound the zeros of Sobolev orthogonal polynomials. The boundedness of the zeros is a consequence of the boundedness of the multiplication operator $Mf(z) = z\,f(z)$; in fact, the zeros of the Sobolev orthogonal polynomials are contained in the disk $\{z : |z| \leq \|M\|\}$ (see [4]).

In [1] there are some answers to the question stated in [3] about some conditions for M to be bounded: the more general result on this topic is [1, Theorem 8.1] which characterizes in a simple way (in terms of equivalent norms in Sobolev spaces) the boundedness of M for the classical "diagonal" case

$$\|q\|_{W^{k,p}(\mu_0,\mu_1,\ldots,\mu_N)} := \left(\sum_{k=0}^{N} \|q^{(k)}\|^p_{L^p(\mu_k)} \right)^{1/p} .$$

In [2] the authors deal with the asymptotic behavior of extremal polynomials with respect to non-diagonal Sobolev norms in a very general case.

In this work we improve the results of [2] in the following context. Since a wide majority of works about Sobolev spaces (both pure and applied) are focused on the case $N = 1$, we

*Research partially supported by three grants from Ministerio de Ciencia e Innovación (MTM 2009-12740-C03-01, MTM 2009-07800 and MTM 2008-02829-E), Spain, and a grant from U.C.III M./C.A.M. (CCG08-UC3M/ESP-4516), Spain.

will assume that this is the situation throughout this work [5]. That is why we consider the weight matrix V as

$$V := \begin{pmatrix} a & b \\ \overline{b} & c \end{pmatrix}, \tag{1}$$

where a, b and c are measurable functions, and V is a positive definite matrix μ-almost everywhere. Then, we define the Sobolev norm

$$\|q\|_{W^{N,p}(V\mu)} := \left(\int (q, q')V(q, q')^* \, d\mu \right)^{1/2}. \tag{2}$$

In [6] the authors study the boundedness of the zeros of the extremal polynomials for this norm under the ellipticity hypothesis $|b|^2 \leq (1-\varepsilon)ac$, μ-almost everywhere for some fixed $0 < \varepsilon \leq 1$.

In this work [5], we are interested in obtaining similar results in the worst case, i.e., when the quadratic form V degenerates in an arbitary set E with $\mu(E) = 0$. In this sense, we replace the useful (but technical) hypothesis $|b|^2 \leq (1-\varepsilon)ac$ (which avoids the degeneracy of the quadratic form V), by a more natural one (in this context) involving integrability properties of the measures. In particular, for the most important case ($p = 2$, corresponding to Sobolev orthogonal polynomials) this new condition is $(c\, d\mu/ds)^{-1} \in L^1(\gamma)$, where $d\mu/ds$ is the Radon-Nykodim derivative of μ with respect to the Euclidean length in γ, the supporting curve of the measure μ.

A particular case (simpler to state) of our main results is the following.

Theorem 1. *Let γ be a finite union of rectifiable compact curves, μ a finite Borel measure with compact support $S(\mu) = \gamma$, and V a positive definite matrix μ-almost everywhere defined as in* (1). *Assume that $\left(c\, d\mu/ds\right)^{-1} \in L^1(\gamma)$ and there exists a constant C such that*

$$c \leq C\, a\,,$$

μ-almost everywhere. Let $\{q_n\}_{n\geq 0}$ be the sequence of Sobolev orthogonal polynomials with respect to $V\mu$. Then the zeros of the polynomials in $\{q_n\}_{n\geq 0}$ are uniformly bounded in the complex plane.

This Theorem also holds for extremal polynomials in the case $1 \leq p \leq 2$ (Soboloev orthogonal polynomials are the extremal polynomials with $p = 2$).

In the context of extremal polynomials there is no such a thing as the usual three term recurrence relation for orthogonal polynomials in L^2. Therefore it is very complicated to find an explicit expression for the extremal polynomial of degree n. Hence, it is especially important to count with an asymptotic estimate for the behavior of extremal polynomials.

As an application of Theorem 1 we can deduce the asymptotic behavior of extremal polynomials. In particular, we obtain the n-th root and the zero counting measure asymptotics both of those polynomials and their derivatives of any order.

Keywords: Sobolev orthogonal polynomials, non-diagonal Sobolev norms, extremal polynomials, asymptotic behavior.

AMS Classification: 41A10, 46E35, 46G10.

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• I Jaen Conference on Approximation Theory
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Recurrence relations and vector equilibrium problems arising from a model of non-intersecting squared Bessel paths

Pablo M. Román and A.B.J. Kuijlaars

Abstract

We consider the model of n non-intersecting squared Bessel processes with parameter α, in the confluent case where all particles start, at time $t = 0$, at the same positive value $x = a$, remain positive, and end, at time $T = t$, at the position $x = 0$. The positions of the paths have a limiting mean density as $n \to \infty$ which is characterized by a vector equilibrium problem. We show how to obtain this equilibrium problem from different considerations involving the recurrence relations for multiple orthogonal polynomials associated with the modified Bessel functions.

We also extend the situation by rescaling the parameter α, letting it increase proportionally to n as n increases. In this case we also analyze the recurrence relation and obtain a vector equilibrium problem for it.

Keywords: Vector equilibrium problem, recurrence relations, squared Bessel process.

AMS Classification: 30C15 .

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Some numerical methods for computing the area of a triangular surface

Paul Sablonnière

Abstract

Given a triangular surface patch defined by the equation $z = f(x, y)$ on the reference triangle $T := \{(x, y) \in \mathbb{R}^2 \,|\, 0 \le x + y \le 1\}$, we approximate its area

$$A := \int_0^1 \int_0^{1-x} \sqrt{1 + \partial_1 f^2 + \partial_2 f^2}\, dxdy, \quad \partial_1 f := \frac{\partial f}{\partial x}, \quad \partial_2 f := \frac{\partial f}{\partial y},$$

by several methods. First, one can use classical interpolatory or Gauss quadrature rules on triangles. Another idea is to approximate the surface by a quadratic one and to compute the area of the latter. We prove that this computation can be done exactly. However, as it is rather complex, we propose a mixed method where the univariate inner integral $A_i(x) := \int_0^{1-x} \sqrt{1 + \partial_1 f^2 + \partial_2 f^2}\, dy$ is done exactly and then numerical methods are used for the outer integral $A := \int_0^1 A_i(x)\, dx$.
More generally, if the function f is defined on a polygonal domain endowed with a given triangulation, it can be approximated by a C^0 or C^1 piecewise quadratic function. Therefore the above methods can be extended to functions defined on such domains.

Keywords: surface area, triangular surface, quadrature rules for surface area.
AMS Classification: 65D30, 65D32.

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On the Favard-type theorem and the Jackson-type theorem

Ryozi Sakai and Noriaki Suzuki

We consider a weighted polynomial approximation for the exponential weight $w = e^{-Q}$ in a class $\mathcal{F}(C^2+)$, where Q is a nonnegative and even function on the real line $\mathbb{R}$. For $1 \leqslant p \leqslant \infty$, $n \in \mathbb{N}$ and $f \in L_{p,w}(\mathbb{R})$, we put

$$E_{p,n}(w; f) := \inf_{P \in \mathcal{P}_n} \|w(f - P)\|_{L_p(\mathbb{R})},$$

where $\mathcal{P}_n$ is the set of all polynomials of degree not more than n. For $r \in \mathbb{N}$ and $\delta > 0$, we set

$$\mathcal{K}_{r,p}(w, f, \delta) := \inf\{\|w(f - g)\|_{L_p(\mathbb{R})} + \delta^r \|wg^{(r)}\|_{L_p(\mathbb{R})}\},$$

where the infimum is taken over $g \in L_{p,w}(\mathbb{R})$ such that $g^{(r-1)}$ is absolutely continuous and $g^{(r)} \in L_{p,w}(\mathbb{R})$. For a weight in a subclass $\mathcal{F}_r(C^2+) \subset \mathcal{F}(C^2+)$, we also define a new modulus of smoothness $\Omega_{r,p}(w, f, \delta)$ as follows:

$$\Omega_{r,p}(w, f, \delta) := \inf_{P \in \mathcal{P}_{r-1}} \omega_{r,p}(w, f - P, \delta).$$

The main result of our talk is the following theorems.

Theorem 1. *Let $w = \exp(-Q) \in \mathcal{F}_r(C^2+)$. Then for every $f \in L_{p,w}(\mathbb{R})$, we have*

$$C_1\Omega_{r,p}(w, f, \delta) \leqslant \mathcal{K}_{r,p}(w, f, \delta) \leqslant C_2\Omega_{r,p}(w, f, \delta), \qquad 0 < \delta < C_3.$$

We denote by a_x the Mhaskar-Rakhmanov-Saff numbers of Q.

Theorem 2. *Let $s \geqslant 0$ be an integer. For any $f \in C^{s-1}(\mathbb{R})$ such that $f^{(s-1)}$ is absolutely continuous and $f^{(s)} \in L_{p,w}(\mathbb{R})$ and any integer $n \geqslant r + s$, we have*

$$(*) \qquad E_{p,n}(w; f) \leqslant C\left(\frac{a_n}{n}\right)^s \mathcal{K}_{r,p}\left(w, f^{(s)}, \frac{a_n}{n}\right).$$

In particular, if $w \in \mathcal{F}_r(C^2+)$, then we have

$$E_{p,n}(w; f) \leqslant C\Omega_{r,p}\left(w, f, \frac{a_n}{n}\right).$$

By the way, we have already known various types of estimates for the degree of weighted approximation $E_{p,n}(w; f)$:

(A) $E_{p,n}(f; w) \leqslant C_1\omega_{1,p}(f, w, C_2\frac{a_n}{n})$ (see [2, 3]),

(B) $E_{p,n}(w; f) \leqslant C_3(\frac{a_n}{n})^s\|wf^{(s)}\|_{L_p(\mathbf{R})}$ (see [4]),

(C) $E_{p,n}(w; f) \leqslant C_4(\frac{a_n}{n})^s\mathcal{K}_{r,p}(w, f^{(s)}, \frac{a_n}{n})$ (the above Theorem 1),

(D) $E_{p,n}(f, w) \leqslant C_5\tilde{\mathcal{K}}_{r,p}(w, f, \frac{a_n}{n}) \sim \omega_{r,p}(w, f, C_6\frac{a_n}{n})$ (see [1, 2, 5]).

We remark here that these inequalities are equivalent in a sense. In fact, (B) is shown by use of (A) (cf. [7]). Using (B) we proved (C). If we use [1, Theorem 1.3 (b)] and [5, Theorem 3.10] we see easily that (C) means (D). Trivially, we have (A) from (D).

Finally, we consider a fractional version of our result $(*)$. Let $w \in \mathcal{F}(C^2+)$ and $1 \leqslant p \leqslant \infty$. Let $1/p \leqslant \beta \leqslant 1$ and g is absolutely continuous with $|g'|^\beta \in L_{p,w}(\mathbb{R})$ (and for $p = \infty$, we require g to be continuous, and gw to vanish at $\pm\infty$). For $f \in L_{p,w}$, we set

$$f^*_{g,\beta}(x) := \sup_{y\in\mathbb{R}} \frac{|f(x) - f(y)|}{|g(x) - g(y)|^\beta}.$$

and we define

$$\mathbf{F}_{g,\beta}(w, p) := \{f \in L_{p,w}(\mathbb{R});\ \|f^*_{g,\beta}\|_{L_\infty} < \infty\}.$$

Theorem 3. *Let $1 \leqslant p \leqslant \infty$ and $1/p \leqslant \beta \leqslant 1$. Then for $f \in \mathbf{F}_{g,\beta}(w, p)$,*

$$E_{n,p}(w, f) \leqslant C(\frac{a_n}{n})^\beta\|w|g'|^\beta\|_{L_p(\mathbb{R})}\|f^*_{g,\beta}\|_{L_\infty(\mathbb{R})}.$$

Corollary 4. *Let $w = \exp(-Q) \in \mathcal{F}(C^2+)$. Let $1 \leqslant p \leqslant \infty$, $1/p \leqslant \beta \leqslant 1$. Then, for $f \in \mathbf{F}_{Q,\beta}(w, p)$*

$$E_{n,p}(w, f) \leqslant CL^{1/p}(\frac{a_n}{n})^\beta\|f^*_{Q,\beta}\|_{L_\infty(\mathbb{R})},$$

where $L := \sup_{x\in\mathbb{R}} w^{p-1/2}(x)|Q'(x)|^{p\beta-1}$.

Keywords: asymptotic formula, shape preserving approximation, simultaneous approximation, saturation.

AMS Classification: 41A28, 41A40, 41A60.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Integer translates of a square integrable function and the periodization function

Sandra Saliani

Abstract

The periodization function of $\psi \in L^2(\mathbb{R})$ is the bracket product

$$p_\psi(\xi) = [\hat{\psi}, \hat{\psi}](\xi) = \sum_{k\in\mathbb{Z}} |\ \hat{\psi}(\xi + k)\ |^2.$$

It is known that many properties of the shift invariant system

$$\mathcal{B} = \{\psi_k, k \in \mathbb{Z}\}, \quad \psi_k(x) = T_k\psi(x) = \psi(x - k),$$

such as orthonormality, basis and frames properties, correspond to properties of p_ψ.

A non-redundancy notion that arises in functional analysis is ℓ^2-linear independence. $\mathcal{B}$ is said to be ℓ^2-linearly independent if there is no non-zero sequence $\{c_n\} \in \ell^2(\mathbb{Z})$ such that

$$\lim_{n\to+\infty} \| \sum_{|k|\leq n} c_k\psi_k\|_{L^2(\mathbb{R})} = 0.$$

We shall focus on the ℓ^2-linear independence of $\mathcal{B}$ and we shall answer the following open conjecture by Guido Weiss [1] : $\mathcal{B}$ is ℓ^2-linearly independent if and only if $p_\psi > 0$ a.e.

Keywords: Shift invariant system, ℓ^2-linear independence, basis, periodization function.
AMS Classification: 42C40, 42A16.

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Refinable ripplets with dilation 3*

Laura Gori, Francesca Pitolli and Elisabetta Santi

Abstract

It is known that refinable functions with dilation M are solutions to the refinement equation

$$\varphi(x) = \sum_{k\in\mathbb{Z}} a_k \, \varphi(Mx - k), \qquad x \in \mathbb{R},$$

where the entries of the *mask* $\mathbf{a} = \{a_k\}_{k\in\mathbb{Z}}$ are real numbers and $M > 1$ is an integer parameter. Refinable functions with dilation $M \geq 3$ have remarkable interest in the applications. For instance, orthogonality and symmetry, which are desirable properties in digital imaging, cannot be achieved by refinable functions with dilation $M = 2$. On the contrary, refinable functions both orthogonal and symmetric were constructed in the case when $M \geq 3$ [1, 4].

On the other hand, in approximation problems and curve/surface design it is important to dispose of refinable bases with shape preserving properties. In this context, a role is played by refinable ripplets, i.e. refinable functions whose integer translates form a totally positive basis (cf. [2, 3] in the case $M = 2$ and $M \geq 3$, respectively).

In this talk, we present a method for constructing refinable ripplets with dilation $M = 3$ and we analyze their approximation properties.

Keywords: Refinable function, total positivity, shape preserving approximation.

AMS Classification: 42C40, 42C15, 65D17.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

New phenomenons in coconvex approximation

Dany Leviatan and Igor A. Shevchuk

Abstract

We will discuss the recent results by G.Dzyubenko, J.Gilewicz and the authors. Say, for the class W^r of $r-1$ times absolutely continuous differential functions, satisfying $|f^{(r)}(x)| \le 1$ for almost all $x \in [-1, 1]$, we have

Theorem 1. *Let $r \ge 3$ and $a \in (-1, 1)$. If $f \in W^r$ and*

$$f''(x)(x-a) \ge 0, \quad x \in [-1, 1],$$

then for each $n \ge N(a, f)$ there is an algebraic polynomial P_n of degree n, such that

$$P_n''(x)(x-a) \ge 0, \quad x \in [-1, 1],$$

and

$$|f(x) - P_n(x)| \le c(r) \left(\frac{1}{n^2} + \frac{1}{n}\sqrt{1-x^2} \right)^r, \quad x \in [-1, 1],$$

where $c(r)$ is a constant, depending only on r.

The number N cannot be taken independent of f. If either $r \le 2$ or a function f changes its convexity more than once, then Theorem 1 can be improved.

Keywords: Shape preserving approximation, coconvex, pointwise.
AMS Classification: 41A10, 41A17, 41A25, 41A29.

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• Úbeda, Jaén, Spain, July 4th-9th, 2010

Approximation Theory analysis for Hermite learning and normal estimation*

Lei Shi, Xin Guo and Ding-Xuan Zhou

Abstract

We consider the problem of learning from data involving function values and gradients in a framework of least-square regularized regression in reproducing kernel Hilbert spaces.The additional data for function gradients improve learning performance of the algorithm. Error analysis is done by means of sampling operators for sample error and integral operators in Sobolev spaces for approximation error. The method for analysis is also applied to another learning theory problem of estimating normals on manifolds from random samples.

Keywords: Learning theory, hermite learning, reproducing kernel Hilbert spaces, representer theorem, sampling operator, integral operator, normal estimation, linear algebra, gradient learning, Riemannian manifold.

AMS Classification: 68T05, 62J02.

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*The work described in this talk is supported partially by the Research Grants Council of Hong Kong [Project No. CityU 104007] and National Science Fund for Distinguished Young Scholars of China [Project No. 10529101].

• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Algebraic and analytical study of a class of vector-valued rational interpolation procedures

Avram Sidi

Abstract

In [1], we presented some new vector-valued rational interpolation procedures for vector-valued functions $F(z)$, $F : \mathbb{C} \to \mathbb{C}^N$. Given the interpolation points ξ_i and the function values $F(\xi_i)$, $i = 1, 2 \ldots$, the interpolants produced by these procedures are all of the simple form

$$R_{p,k}(z) = \frac{U_{p,k}(z)}{V_{p,k}(z)} = \frac{\sum_{j=0}^{k} c_j\, \psi_{1,j}(z)\, G_{j+1,p}(z)}{\sum_{j=0}^{k} c_j\, \psi_{1,j}(z)},$$

where

$$\psi_{m,n}(z) = \prod_{r=m}^{n} (z - \xi_r), \quad n \geq m \geq 1; \quad \psi_{m,m-1}(z) = 1, \quad m \geq 1,$$

and $G_{m,n}(z)$ is the vector-valued polynomial of interpolation to $F(z)$ at the points ξ_i, $m \leq i \leq n$. The c_i are scalars, and they are determined in different ways by the three methods. As such, $R_{p,k}(z)$ interpolates $F(z)$ at ξ_i, $1 \leq i \leq p$.

In this lecture, we review the developments that have taken place following those of [1]. We first review the algebraic properties of these interpolants as presented in [2]. Next, we discuss some of the convergence properties of two of the methods, as they are being applied to functions $F(z)$ meromorphic with simple poles in a domain. Choosing the interpolation points appropriately, we derive de Montessus type convergence results for the interpolants and König type convergence results for the poles and residues. The study in [3] pertains to the method denoted IMMPE, whereas that in [4] and [5] pertains to the method denoted IMPE. The IMPE approximations are defined via a least-squares procedure, and they exhibit different behavior, depending on whether the residues of $F(z)$ at its poles are mutually orthogonal or not.

Keywords: Vector-valued rational interpolation, simultaneous approximation, de Montessus theorem, König theorem, asymptotic expansions, meromorphic functions.

AMS Classification: 30E10, 41A05, 41A20, 41A28, 41A60.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Abel-type theorems for rational series*

Mehrdad Simkani

Abstract

The classical theory of power series began with Abel's theorem on the circle of convergence. Here we consider Abel-type theorems for series with rational functions as terms. We do allow the number of finite poles to approach infinity. The mathematics we consider here belongs to Leja and Walsh schools of thought.

Keywords: Rational series, power series, circle of convergence.

AMS Classification: 30E10, 40A30, 31A05.

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*Supported by the Office of Research, UM-Flint.

Asymptotics of orthogonal polynomials for a non analytic weight*

Vítor L. Sousa, A. Foulquié Moreno and A. Martínez-Finkelshtein

Abstract

We consider orthogonal polynomials on $[-1, 1]$ with respect to a generalized Jacobi weight modified by a step-like function:

$$(1-x)^{\alpha}(1+x)^{\beta}|x_0-x|^{\gamma} \times \begin{cases} 1, & \text{for } x \in [-1, x_0), \\ c^2, & \text{for } x \in [x_0, 1], \end{cases}$$

with $c > 0$, $\alpha, \beta, \gamma > -1$, and $x_0 \in (-1, 1)$. We obtain strong uniform asymptotics in the whole plane and show explicitly the first two terms of the asymptotic expansion for the monic orthogonal polynomials. The proof is based on the steepest descendent method of Deift and Zhou applied to the non-commutative Riemann-Hilbert problem characterizing the orthogonal polynomials. A feature of this situation is that the local analysis at x_0 has to be carried out in terms of confluent hypergeometric functions.

Keywords: Orthogonal polynomials, asymptotics, Riemann-Hilbert analysis, confluent Hypergeometric functions.

AMS Classification: 42C05, 41A60, 33C15.

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*FCT - MCTES *Portugal* contract/grant sfrh/bd/29731/2006.

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On the interplay between asymptotic and numerical methods to solve differential equations problems

Renato Spigler

Abstract

Asymptotic and numerical methods usually represent to independent approaches to solve applied mathematical problems. Here we describe some cases when they are used simultaneously, in a complementary way.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

The Asymptotic Distribution of Poles of AAK Approximants

Herbert Stahl, Laurent Baratchart and Maxim Yattselev

Abstract

We are concerned with the asymptotic behavior of AAK (Adamjan-Arov-Krein) approximants to functions f that are holomorphic outside the unit disk $\mathbb{D}$ and a little bit beyond, and have all their singularities in a compact set of capacity zero in $\mathbb{D}$. Among these singularities there should be also branch points. Our main object is the investigation of the asymptotic distribution of the poles of the AAK approximants, and we address the principles that determine the distribution and its support, which typically consists of a union of analytic arcs in $\mathbb{D}$.

AAK approximants are meromorphic functions in $\mathbb{D}$ with a controlled, finite number of poles, and they have a minimal deviation in the uniform norm on $\mathbb{T}$ from the function f to be approximated. There is an important and very interesting connection with Hankel operators and their singular values, which is central for their understanding, but in our talk this topic will play no significant role, instead we shall concentrate on questions that are important for understanding of the analytic background of the arcs in $\mathbb{D}$ that form the support of the asymptotic distribution. Methodologically, the main tools of the investigation belong to potential theory and geometric theory of functions.

Keywords: AAK-approximation, asymptotic distribution of poles of AAK-approximants, potential theory, extremal domains.

AMS Classification: (primary) 41A20, 41A21, 41A30, 42C05; (secondary) 30D50, 30D55, 30E10, 31A15.

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On Markov, Bernstein and Chebyshev type inequalities for k-monotone polynomials*

András Kroó and József Szabados

Abstract

In a recent joint paper we initiated the study of Markov type inequalities for the so called k**-monotone** polynomials, whose first k derivatives are nonnegative in the interval considered. For the first derivative of k-monotone polynomials the exact constant of Markov type inequality was found for uniform and L_1 norms. This exact constant was given in terms of the largest zeros of certain Jacobi polynomials. Moreover it was shown that in general for derivatives of order j the sharp order of magnitude of Markov constants is $\left(\frac{n^2}{k}\right)^j$.

Subsequently we also established asymptotically sharp Bernstein type inequalities for derivatives of k-monotone polynomials, when their size is measured on $[-1,1]$ in weighted uniform or L_1 norms with weight $\varphi(1-x)$, where φ is such that $\varphi(x)/x$ is decreasing. It turns out that the order of magnitude of constants in Bernstein type inequalities for derivatives of order j is $\left(\varphi\left(\frac{k}{n^2}\right)\frac{n^2}{k}\right)^j$.

In addition we also give an exact solution to the Chebyshev type extremal problem of finding monic k-monotone polynomials of minimal uniform and L_1 norms. Just as in the case of Markov type inequalities for k-monotone polynomials these extremal polynomials are related to certain Jacobi polynomials.

Keywords: Markov-, Bernstein- and Chebyshev type inequalities, k-monotone polynomial, exact order.

AMS Classification: 41A17.

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*Participation supported for both authors by OTKA Grant No. T77812

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Product integration based on quasi-interpolation for boundary integral equations*

P. Sablonnière, D. Sbibih and M. Tahrichi

Abstract

We consider the numerical solution of boundary integral equation of the second kind, that are reformulations of Laplace's equation $\Delta u = 0$ on connected regions D in R^2 with a smooth boundary Γ. Symbolically, we write the boundary integral equation as $(\lambda - \mathcal{K})\rho = g$. The numerical method is the product integration method based on a periodic cubic discrete spline quasi-interpolant in the parametrization variables. We approximate both the curve Γ and the unknown solution ρ by using such quasi-interpolant.

Keywords: Integral equation, product integration, quasi-interpolant.

AMS Classification: 41A05, 45A05, 45B0, 65D05, 65D07.

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On approximation properties of Shannon sampling operators with averaged and dilated bandlimited kernels*

Gert Tamberg

Abstract

In this talk we consider some generalized Shannon sampling operators, which are defined by band-limited kernels. For the uniformly continuous and bounded functions $f \in C(\mathbb{R})$ the generalized sampling series (see [2], [1] and references cited there) with a kernel function $s \in L^1(\mathbb{R})$ are given by ($t \in \mathbb{R}$; $W > 0$)

$$(S_W f)(t) := \sum_{k=-\infty}^{\infty} f(\frac{k}{W}) s(Wt-k), \text{ where } \sum_{k=-\infty}^{\infty} s(u-k) = 1. \tag{1}$$

In this talk we study an even band-limited kernel s, defined as the Fourier transform of an even window function $\lambda \in C_{[-1,1]}$, $\lambda(0) = 1$, $\lambda(u) = 0$ ($|u| \geqslant 1$) by the equality $s(t) := s(\lambda; t) := \sqrt{\frac{\pi}{2}} \lambda^\wedge(\pi t)$. Here the kernel function s belongs to a Bernstein class B^1_π of functions of exponential type provided $s \in L^1(\mathbb{R})$. Many kernels are represented in this form, see [3, 4, 5].

We studied in [5] an dilated kernel $s_\alpha(t) = \alpha s(\alpha t)$. In the case of $s \in B^1_\pi$ for $0 < \alpha \leqslant 2$ we get an sampling operator $S_{W,\alpha} : C(\mathbb{R}) \to B^\infty_{\alpha\pi W} \subset C(\mathbb{R})$.

We can define an averaged kernel in form $\overline{s}_m(t) := \frac{1}{m} \int\limits_{-m/2}^{m/2} s(t+v)\, dv$. If $s \in B^1_\pi$ then also $\overline{s}_m \in B^1_\pi$ (see [4] Th. 3).

We consider in this talk the following problems for the generalized sampling operators $S_W : C(\mathbb{R}) \to C(\mathbb{R})$:

1) To calculate the operator norms

$$\|S_W\| = \sup_{u\in\mathbb{R}} \sum_{k=-\infty}^{\infty} |s(u-k)|; \tag{2}$$

*This research was supported by the Estonian Science Foundation, grants 6943, 7033, and by the Estonian Min. of Educ. and Research, project SF0140011s09.

2) To estimate the order of approximation

$$\|f - S_W f\|_C \leqslant M\omega_k(f, \frac{1}{W}) \tag{3}$$

in terms of the k-th modulus of smoothness $\omega_k(f, \delta)$.

Keywords: Sampling operators, band-limited kernels, dilated kernels, averaged kernels.
AMS Classification: 41A25, 41A45, 42A24.

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Recent developments on approximation theory with respect to representative product systems

Rodolfo Toledo

Abstract

A natural generalization of the Vilenkin groups is the complete product of arbitrary groups, non necessarily commutative groups (see [1]). In this case we use representation theory in order to obtain orthonormal systems, taking the finite product of the normalized coordinate functions of the continuous irreducible representations appeared in the dual object of the finite groups. These systems are named representative product systems and they are orthonormal and complete systems in L^1, but not necessary uniformly bounded. Representative product systems can be represented on the interval $[0, 1]$, where this systems are also orthonormal under the Lebesgue measure (see [2]).

This work relates the recent results I obtained in this topic. It also deals with the differences between a Vilenkin system and a general representative product system based in the comportment of Dirichlet kernels.

Keywords: Walsh system, Vilenkin system, Fourier series, Dirichlet kernels, Lebesgue constants, representative product systems.

AMS Classification: 42C10.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

Applications of the monotonicity of extremal zeros of orthogonal polynomials in interlacing and optimization problems

Ferenc Toókos and Wolfgang Erb

Abstract

We investigate monotonicity properties of extremal zeros of orthogonal polynomials depending on a parameter. Using a functional analysis method we prove the monotonicity of extreme zeros of associated Jacobi, associated Gegenbauer and q-Meixner-Pollaczek polynomials. We show how these results can be applied to prove interlacing of zeros of orthogonal polynomials with shifted parameters and to determine optimally localized polynomials on the unit ball.

Keywords: Monotonicity of zeros, associated Jacobi polynomials, associated Gegenbauer polynomials, q-Meixner-Pollaczek polynomials, interlacing of zeros, orthogonal polynomials on the unit ball.

AMS Classification: 33C47, 33C55, 33D15.

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The direct and inverse spectral problems for a class of non-self-adjoint Jacobi matrices*

Mikhail Tyaglov

Abstract

We solve the direct and inverse spectral problems for finite tridiagonal matrices of the form

$$J_n^{(k)} = \begin{pmatrix} a & b_1 & 0 & \dots & 0 & 0 \\ c_1 & 0 & b_2 & \dots & 0 & 0 \\ 0 & c_2 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & b_{n-1} \\ 0 & 0 & 0 & \dots & c_{n-1} & 0 \end{pmatrix}, \tag{1}$$

where $a \in \mathbb{R}$, $c_j > 0$ and, for some number k, $0 \leqslant k \leqslant n-1$,

$$b_j < 0 \qquad \text{whenever} \quad j \leqslant k,$$

$$b_j > 0 \qquad \text{whenever} \quad j > k.$$

It is established that for any k, $0 \leqslant k \leqslant n-1$, the characteristic polynomial of the matrix $J_n^{(k)}$ is a generalized Hurwitz polynomial [1]. Conversely, for any generalized Hurwitz polynomial p of degree n, there exists a unique matrix of the form (1) whose characteristic polynomial is p. Special cases of $k = 0$ and $k = n-1$ are particularly discussed.

Keywords: Inverse problems, Jacobi matrices, generalized Hurwitz polynomials.
AMS Classification: 47B36.

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• I Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 4th-9th, 2010

On approximation of smooth functions by eigenfunctions of optimal differential operators with constant coefficients*

Vesselin Vatchev

Abstract

For a real-valued function $f \in C^{(n)}(I)$, defined on a finite interval I, and a differential operator $L_n(f) = \sum_{k=0}^{n} a_k f^{(k)}$, with constant coefficients $a_k \in \mathbb{R}$, and eigenfunctions $e^{\lambda_k t}$, λ_k-complex numbers, we obtain the following Jackson's type estimate for approximation of f on I by linear combinations of the form $S_n(f,t) = \sum_{k=1}^{n} b_k e^{\lambda_k t}$.

$$\min_{S_n \in \{\sum_{k=1}^{n} b_k e^{\lambda_k t} |\ a_k \in \mathbb{R}\}} \|f^{(m)} - S_n^{(m)}(f,t)\|_\infty \leq \frac{|I|^{1/q} e^{2|\lambda_n||I|}}{\pi |a_n| 2^{n-2m-1+1/q} |\lambda_n|^{n-m-1}} \|L_n(f)\|_p, \qquad (1)$$

where $|\lambda_n| = \max_k |\lambda_k|$, $m \leq n-1$ is an integer, $p, q \geq 1$, and $\frac{1}{p} + \frac{1}{q} = 1$.

For the particular operator $M_n(f) = f + 1/(2n)! f^{(2n)}$ the rate of approximation by the eigenvalues of M_n for non-periodic analytic functions on a finite interval is established to be exponential. Algorithms and numerical examples are discussed.

Keywords: Approximation of smooth functions, eigenfunctions of differential operators, exponential rate of convergence.

AMS Classification: 94A12, 34B24, 41A58.

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Convergence of spline interpolation processes*

Yuri S. Volkov

Abstract

In 1963 I. Schoenberg has stated the problem on finding conditions at which the derivatives of interpolating splines will converge to corresponding derivatives of function under interpolation. Namely for $f \in C^k$ let s be a spline of the odd degree $2n-1$ which interpolates f on a mesh Δ. What restrictions on mesh sequence $\{\Delta\}$ are needed for $\|s^{(k)} - f^{(k)}\| \to 0$ as $\|\Delta\| \to 0$?

This problem has turn out not such easy even for cubic splines, and it has been solved by efforts of many researchers for a cubic case basically by 1977. For splines of any degree C. de Boor [1] in 1975 has shown, that at $k = 0, 1, \ldots, n-2$ convergence without restrictions on meshes is impossible, he has put forward conjecture what only two middle derivatives ($k = n-1$ and $k = n$) can converge without any restrictions, and restrictions on $\{\Delta\}$ too are necessary for convergence of the high-order derivatives. In 1984 by the author [2] had been constructed examples of divergent processes for $k = n+1, \ldots, 2n-1$. And in 2001 A. Shadrin [3] proved unconditional convergence for $k = n$.

We have established, that for convergence $S^{(k)}$ to $f^{(k)}$ as $f \in C^k$ the restrictions on sequence of meshes $\{\Delta\}$ should coincide with the restrictions on $\{\Delta\}$ for convergence $S^{(2n-1-k)}$ to $f^{(2n-1-k)}$ as $f \in C^{2n-1-k}$. That completes the proof of C. de Boor's conjecture [1], i. e. $S^{(k)}$ converges to $f^{(k)}$ at $k = n-1$ for any mesh sequence [4], [5].

The developed approach is applied also to research of convergence of interpolation processes for even degree splines. The connection between different constructions of splines on M. Marsden and on Yu. Subbotin and connection between the conditions of convergence of interpolation processes is established.

Keywords: spline interpolation, convergence, de Boor's conjecture, B-splines, norm of projector.

AMS Classification: 41A05, 41A15, 41A25.

*The work was partially supported by the Department of Mathematical Sciences of RAS (project 2009-1.3.8), Interdisciplinary integration projects of SB RAS (project 2009-81) and SB RAS with UrB RAS (project 2009-14).

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