III Jaen Conference on Approximation Theory

Úbeda, Jaén, Spain, July 15th-20th, 2012

Approximation Theory, Computer Aided Geometric Design, Numerical Methods and Applications

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Introduction

The Jaen Conference on Approximation Theory series is a new activity of the Jaen Approximation Project. Jaen Approximation Project has organized ten editions of the Ubeda Meeting on Approximation and issues the Jaen Journal on Approximation. The objective of these conferences is to provide a useful and nice forum for researchers in the subjects to meet and discuss. In this sense, the conference program has been designed to keep joined the group during five days with a program full of scientific and social activities. The Conference will be devoted to some significant aspects on Approximation Theory, Computer Aided Geometric Design, Numerical Methods and the Applications of these fields in other areas.

The Conference will take place in Úbeda, a World Heritage Site. Around 50 participants from 14 countries will attend the conference. The Conference features six invited speakers who will give one-hour plenary lectures. Researchers were invited to contribute with a talk or a poster. We have scheduled 16 short talks and a poster session. We will not prepare Proceedings of the Conference, but all the participants are invited to submit manuscripts to the Jaen Journal on Approximation.

Finally, but also important, the Conference provides to participants the possibility to visit World Heritage Sites and taste a wide culinary variety. We will do all the best for accompanying people to enjoy the Conference. We are grateful to all those who have made this project a reality; the University of Jaén (Vicerrectorado de Investigación, Facultad de Ciencias Experimentales, Departamento de Matemáticas), Junta de Andalucía, Diputación Provincial de Jaén, Obra Social Unicaja, Centro Asociado de la UNED de la provincia de Jaén, Ayuntamiento de Úbeda, Ayuntamiento de Cazorla, Parador de Úbeda and Hotel María de Molina. Here we emphasize our commitment to keep on working to improve our university and our province.

The Organizing Committee

Scientific Program

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Poster Session

June 17th-Thursday, 12:00–12:30, UNED (Úbeda).

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Invited Lectures

Short Talks/Posters

Invited Lectures

L^p-approximation with quasi-interpolation by radial basis functions

Martin Buhmann and Feng Dai

Abstract

We consider radial basis function approximations using a localisation of the basis functions known as quasi-interpolation (to be contrasted to the plain linear combinations of shifts of radial basis functions or for instance cardinal interpolation). Using these quasi-interpolants we derive various L^p-error estimates for $p \in [1, \infty]$, including approximations where the approximands are from "rougher" spaces than the approximants. We also consider compression-type approximations from the same linear spaces.

Keywords: radial basis functions, quasi-interpolation.

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On the L_∞-norm of the L_2-spline projector. A short proof of de Boor's conjecture

Manfred v.Golitschek

Abstract

In 2001, A. Yu. Shadrin published a proof of a famous problem in univariate spline theory, well-known as de Boor's conjecture (1973):

> Let $\Delta : t_0 < t_1 < \cdots < t_n$ be a finite sequence of knots. Let $\mathcal{S}_m(\Delta)$ be the linear space of polynomial splines of degree m with simple knots Δ. The L_∞-norm of the L_2 projector $\mathcal{P} := \mathcal{P}_m(\Delta) : C[t_0, t_n] \to \mathcal{S}_m(\Delta)$ is defined by
>
> $$\|\mathcal{P}\|_\infty := \sup \Big\{ \|\mathcal{P}f\|_\infty : \ f \in C[t_0, t_n], \ \|f\|_\infty = 1 \Big\},$$
>
> with respect to the uniform norm $\|\cdot\|_\infty$ on $[t_0, t_n]$.
>
> **Theorem A** (Shadrin). *There exists a positive number K_m depending only on m such that $\|\mathcal{P}\|_\infty \leq K_m$.*

It is the purpose of my talk to provide a short proof of Theorem A.

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Some approximation problems in the biomedical optics*

Andrei Martínez-Finkelshtein and Darío Ramos-López

Abstract

The medical imaging benefits from the advances in constructive approximation, orthogonal polynomials, Fourier and numerical analysis, statistics and other branches of mathematics. At the same time, the needs of the medical diagnostic technology pose new mathematical challenges. This talk outlines a few problems, some of them related to approximation theory, that have appeared in our collaboration with ophthalmologists, although it will contain more questions than answers.

Keywords: approximation theory, orthogonal polynomials, surface fitting, radial basis functions, vision science.
AMS Classification: 41A45, 41A63, 42C05.

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*Insert here, if there exist, the organizations that support your research.

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• III Jaen Conference on Approximation Theory

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On the explicit constructions of weight functions for Chebyshevian spaces and splines

Marie-Laurence Mazure

Abstract

There exists a famous procedure to build Extended Chebyshev spaces by means of weight functions and associated generalised derivatives. Let us recall two well-known facts concerning the procedure in question:

– two different systems of weight functions may lead to the same Extended Chebyshev space;

– on a given closed bounded interval this procedure yields all Extended Chebyshev spaces.

These two statements naturally induced the following question: *an Extended Chebyshev space on a closed bounded interval being given, can we identify all systems of weight functions which can be associated with it via the latter procedure?*

Only recently this question was solved, blossoms and their properties being strongly involved in the proof. I actually gave a talk on this result in the first Jaén Conference in 2010. As a conclusion to my talk, I then mentioned that this nice theoretical result was strongly connected with many other interesting issues, such as the construction of Bernstein-type operators based on Extended Chebyshev spaces. However, at that time I was not yet aware of all its implications, in particular in association with a similar result for splines. The present talk will survey some of these implications.

Keywords: extended Chebyshev spaces, generalised derivatives, Bernstein and B-spline bases, Bernstein and Schoenberg operators, rational spaces, rational splines, blossoms, shape preservation.

AMS Classification: 41A15, 41A36, 65D07, 65D17.

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Spline spaces on meshes with hanging vertices

Larry L. Schumaker

Abstract

We discuss polynomial spline spaces defined on triangular, rectangular, and mixed meshes with hanging vertices. Meshes with hanging vertices are particularly important for applications since refinement is greatly simplified and the resulting spline spaces are much more flexible than traditional splines.

In this talk we describe dimension formulae, the construction of explicit basis functions, their support properties and stability, and the approximation power of the spaces. These spline spaces are especially useful for interpolation, data fitting and the numerical solution of boundary value problems by the finite element method.

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Approximation by multivariate incomplete polynomials*

József Szabados

Abstract

We consider multivariate incomplete polynomials on certain 0-symmetric starlike domains. Polynomial density and quantitative order of approximation will be investigated.

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*Joint work with András Kroó.

Short Talks/Posters

A probabilistic approach to the Hurwitz and Riemann zeta functions*

José A. Adell

Abstract

Let

$$\zeta(s,a)=\sum_{k=0}^{\infty}\frac{1}{(k+a)^s},\quad \mathrm{Re}(s)>1,\quad \mathrm{Re}(a)>0,$$

be the Hurwitz zeta function. The Laurent expansion of $\zeta(s,a)$ about its simple pole at $s=1$ is given by

$$(s-1)\zeta(s,a)=1+\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\gamma_n(a)(s-1)^{n+1},\quad s\in\mathbb{C},\quad \mathrm{Re}(a)>0,$$

where the constants $(\gamma_n(a))_{n\geq 0}$ are known as generalized Stieltjes constants. For $a=1$, $\zeta(s,1)=\zeta(s)$ is the Riemann zeta function and the constants $\gamma_n(1)=\gamma_n$ are called Stieltjes or generalized Euler constants, since γ_0 is the Euler-Mascheroni constant.

In this talk, we give explicit upper estimates for $|\gamma_n|$ useful for large values of n, whose order of magnitude, as $n\to\infty$, is

$$\exp\left\{n\log\log n-\frac{n}{2\log^2 n}\left(1+O\left(\frac{1}{\log n}\right)\right)\right\}.$$

This order of magnitude is close to the leading asymptotic form of the constants γ_n, recently obtained by Knessl and Coffey (2011). On the other hand, for each $n=1,2,...$ and $\mathrm{Re}(a)>0$, we obtain estimates of the form

$$|\gamma_{n-1}(a)-\tau_{n,a}(m)|\leq b_{n,a}(m),$$

where the main term of the approximation $\tau_{n,a}(m)$ is a finite sum involving Bernoulli numbers, and the upper bound $b_{n,a}(m)$ is explicit and has order of magnitude

$$(m+|a|+1)^{n+1}\left(\frac{\alpha}{2\pi}\right)^{m+Re(a)-1},$$

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where $\alpha = 1.615395\cdots$. Numerical computations for γ_0 and γ_1 suggest that the approximation above has, in fact, a geometric rate of decay. Finally, we approximate the Riemann zeta function $\zeta(s)$ by a finite sum given in terms of negative binomial probabilities at a geometric rate of convergence.

To prove the aforementioned results, we use a probabilistic approach based on a differential calculus for linear operators represented by stochastic processes, in particular, gamma and negative binomial processes.

Keywords: Bernoulli number, Hurwitz zeta function, gamma process, linear operator, negative binomial process, Riemann zeta function, Stieltjes constant.
AMS Classification: 60F05, 41A17.

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An inequality of Duffin-Schaeffer-type for Hermite polynomials*

Alexander Alexandrov and Geno Nikolov

Abstract

Let H_n be the n-th Hermite polynomial and a_m be the rightmost zero of H_m, $m \in \mathbb{N}$. Let π_n^r be the set of all algebraic polynomials of degree not exceeding n and having only real coefficients. We prove that if $f \in \pi_n^r$ satisfies $|f| \leq |H_n|$ at the zeros of H_{n+1} then, for $k = 1, \ldots, n$,

$$|f^{(k)}(x+iy)| \leq |H_n^{(k)}(a_{n+1}+iy)| \quad \text{for every} \quad (x,y) \in [-a_{n+1}, a_{n+1}] \times \mathbb{R}$$

and the equality occurs if and only if $f = \pm H_n$.

This result may be viewed as an analog of the famous extension of the classical inequality of Markov, found by Duffin and Schaeffer.

Keywords: Hermite polynomials, Jensen inequalities, Markov inequality, Duffin and Schaeffer type inequalities.

AMS Classification: 41A28, 41A40, 41A60.

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On some convergence criteria for nets of positive operators on continuous function spaces

Francesco Altomare

Abstract

In the spirit of the Korovkin-type Approximation Theory, in [1] we stated some conditions under which nets (or generalized sequences) of positive linear operators acting on the space $C(X)$, X compact, converge to a given positive linear projection T on $C(X)$ (see also [4, Theorem 3.3.3], [2, Theorem 10.3]).

This result has been successfully used in order to investigate the convergence toward T of suitable Feller semigroups on $C(X)$ as well as sequences of iterates of Bernstein-Schnabl operators, Stancu-Schnabl operators and Lototsky-Schnabl operators associated with T (see, for instance, [4, Sections 6.1 and 6,2]).

The talk will be devoted to discuss a new criterion concerning the convergence of nets of positive linear operators on $C(X)$ toward a positive linear operator. The proof of the result is very elementary and the main assumptions refer to the subset of interpolation points of the limit operator and its possible representation by means of suitable functions that always exist provided X is metrizable.

This result not only generalizes the previous one concerning positive projections (providing a new simple proof) but it also allows to characterize the convergence of the iterates of a Markov operator toward a given Markov projection again in terms of the involved interpolation sets.

We finally discuss some applications concerning, in particular, the asymptotic behaviour of the iterates of Bernstein-Schnabl operators on a convex compact subset (not necessarily associated with a positive projection) and the iterates of a special linear operator defined in the setting of compact subsets of normed spaces that generalizes the so-called Cesàro operator on $C([0, 1])$.

All the results are taken from [3].

Keywords: net of positive operators, iterate of Markov operator, Choquet boundary, interpolation set, positive projection, Bernstein-Schnabl operator, Cesàro operator.

AMS Classification: 41A36, 47B65, 47B38.

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Approximation results with respect to multidimensional φ-variation*

Laura Angeloni and Gianluca Vinti

Abstract

The φ-variation in the sense of Musielak-Orlicz [5] is an important extension of the classical concept of variation. In [3] a new generalization of φ-variation in the multidimensional setting was introduced and approximation results were studied for linear convolution integral operators. All the theory was generalized to the nonlinear case in [2]. By means of this definition of φ-variation, inspired by the Tonelli approach, in [4] we also study the problem in the homothetic case, that is, we obtain approximation results for the following family of linear integral operators of homothetic type

$$(T_w f)(\mathtt{s}) = \int_{(\mathbb{R}_0^+)^N} K_w(\mathtt{t}) f(\mathtt{st})\, d\mathtt{t}, \quad w > 0, \quad \mathtt{s} \in (\mathbb{R}_0^+)^N;$$

here $\{K_w\}_{w>0}$ is a family of approximate identities and $f \in BV^{\varphi}((\mathbb{R}_0^+)^N)$, where $BV^{\varphi}((\mathbb{R}_0^+)^N) := \{f \in L^1((\mathbb{R}_0^+)^N) : \exists \lambda > 0 \text{ such that } V^{\varphi}[\lambda f] < +\infty\}$ denotes the space of functions with bounded φ-variation on $(\mathbb{R}_0^+)^N$. In particular, using a convergence result for the φ-modulus of smoothness in the multidimensional frame [1], we prove that there exists $\mu > 0$ such that

$$\lim_{w \to +\infty} V^{\varphi}[\mu(T_w f - f)] = 0,$$

if f is φ-absolutely continuous. Results concerning the rate of approximation are also given.

Keywords: convolution integral operators, homothetic integral operators, multidimensional φ-variation, rate of approximation, Lipschitz classes, φ-modulus of smoothness.
AMS Classification: 26B30, 26A45, 41A25, 41A35.

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• III Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 15th-20th, 2012

A direct approach to the asymptotic behaviour of general Mellin convolution operators*

Carlo Bardaro and Ilaria Mantellini

Abstract

The theory of Mellin transform, along with its various applications to boundary value problems in wedge-shaped domains, was widely developed during the last century. As in Fourier theory, which is strictly connected with the Mellin one, a key role is played by the convolution integrals over the positive real line multiplicative group, endowed with its natural Haar measure $d\mu(t) = t^{-1}dt$. In Mellin frame, these operators are defined by

$$(T_w f)(s) = \int_0^{+\infty} K_w(t) f(ts) \frac{dt}{t}, \quad s \in \mathbb{R}^+, \ w > w_0,$$

where f belongs to the domain of T_w. In 1991, R.G. Mamedov wrote an interesting, but unfortunately not widely diffused, monograph in which he exposed the Mellin transform theory and its applications in approximation theory, using an approach fully independent of the classical Fourier analysis, by working with a "logarithmic" version of the notions of uniform continuity, modulus of continuity, derivatives, known as "Mellin derivatives". Later on, another fundamental contribution in this direction was given in 1997 by P. L. Butzer and S. Jansche, in which, with the same approach, approximations by sequences or nets of Mellin convolution operators are studied in L^p spaces. Our aim is to continue this research by considering asymptotic formulae, in particular of Voronovskaya type, for sequences of Mellin convolution operators, their suitable linear combinations and iterates in order to obtain better order of pointwise and uniform convergence.

Keywords: asymptotic formula, Mellin convolution operators, linear combinations, iterates.
AMS Classification: 41A35, 41A25, 47G10.

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Mixed hyperbolic/trigonometric spaces for design

Martine Brilleaud and Marie-Laurence Mazure

Abstract

We investigate the class of five-dimensional null spaces of linear differential operators with constant coefficients and odd characteristic polynomials. One of the advantages of this class is that it permits to mix trigonometric and hyperbolic functions within the same space, and we more specially focus on this interesting blending.

We determine the corresponding critical length for design [3], that is, the maximal positive number ℓ such that, on any interval $[a, b]$ with $b - a < \ell$, existence of blossoms is guaranteed [8]. Equivalently, existence of Bernstein bases is guaranteed on $[a, b]$ if and only if for $b - a < \ell$, such bases being then automatically the optimal normalised totally positive bases. This is due to the fact that the presence of blossoms permits the development of all the classical design algorithms which are automatically corner cutting.

The interest of this class of spaces for geometric design is illustrated with corresponding Bézier and L-spline curves. Beforehand, we have to determine when such L-splines [9, 5] can be used for design. This is made possible via a recently established criterion, see [7, 2].

Keywords: extended Chebyshev spaces, weight functions, generalised derivatives, Bernstein bases, B-spline bases, L-splines, total positivity, optimal bases, blossoms.
AMS Classification: 65D07, 65D17.

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Multivariate Sampling Kantorovich operators: approximation results*

Danilo Costarelli and Gianluca Vinti

Abstract

In [3] some approximation results for function of several variables by means of Sampling Kantorovich operators are given. The operators taken into consideration extend to the multivariate setting, those studied in [1] and are of the form:

$$(S_w^\chi f)(\underline{x}) \;=\; \sum_{\underline{k}\in\mathbb{Z}^n} \chi(w\underline{x}-t_{\underline{k}}) \left[\frac{w^n}{A_{\underline{k}}}\int_{R_{\underline{k}}^w} f(\underline{u})\, d\underline{u}\right] \qquad (\underline{x}\in\mathbb{R}^n,\ w>0), \qquad \text{(I)} \qquad (1)$$

where $\chi:\mathbb{R}^n\to\mathbb{R}$ is a kernel function satisfying suitable assumptions, $t_{\underline{k}}=(t_{k_1},...,t_{k_n})$ is such that $(t_{k_i})_{k_i\in\mathbb{Z}}$, $i=1,2,...,n$ are suitable sequences of real numbers and $A_{\underline{k}}=\Delta_{k_1}\cdot\Delta_{k_2}\cdot ...\cdot\Delta_{k_n}$ with $\Delta_{k_i}=t_{k_i+1}-t_{k_i}$, $i=1,2,...,n$. Moreover,

$$R_{\underline{k}}^w \;=\; \left[\frac{t_{k_1}}{w},\frac{t_{k_1+1}}{w}\right]\times\left[\frac{t_{k_2}}{w},\frac{t_{k_2+1}}{w}\right]\times ...\times\left[\frac{t_{k_n}}{w},\frac{t_{k_n+1}}{w}\right] \qquad (w>0).$$

Finally let $f:\mathbb{R}^n\to\mathbb{R}$ be a locally integrable function such that the above series (1) converges for every $\underline{x}\in\mathbb{R}^n$. The multivariate setting of such operators allows us to study applications to image processing. The main results concern both the pointwise and uniform convergence and the modular convergence, for the family $(S_w^\chi f)_{w>0}$ where, in the latter case, f belongs to an Orlicz space [2]. The general approach in Orlicz spaces allows us to obtain, in particular, results in L^p-spaces, Zygmund spaces and exponential spaces.

Keywords: sampling Kantorovich operators, Orlicz spaces, irregular sampling, modular convergence, $L^\alpha\log^\beta L$-spaces, exponential spaces.
AMS Classification: 41A35, 46E30, 47A58, 47B38, 94A12, 94A20.

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Linear-phase moments in approximation theory and wavelet analysis*

Bin Han

Abstract

Approximation order of a shift invariant space generated by a single generating function is linked to the Strang-Fix condition, while the order of Linear-pase moments of the generating function controls how the polynomials are exactly reproduced by the integer shifts of the generating function. It turns out that linear-phase moments also play an interesting role in wavelet analysis. In this talk we shall discuss several applications of linear-phase moments in wavelet analysis: (1) Subdivision schemes with linear-phase moments which produce nearly shifted interpolatory subdivision schemes, (2) Role of linear-phase moments in the construction of symmetric tight framelets and the frame approximation order of dual wavelet frames (3) Linear-phase moments in the design of orthogonal filters used the Dual-Tree Complex Wavelet Transform, which significantly outperforms the commonly used tensor product wavelets in signal and image processing.

Keywords: linear-phase moments, approximation order, wavelets, subdivision schemes.
AMS Classification: 42C40.

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Markov-Nikolskii type inequalities for monotone polynomials of order k

Oleksiy Klurman

Abstract

Let $M_{q,p}(n,k) := \sup_{P_n \in \mathbb{P}_n} \frac{\|P_n^{(k)}\|_{L_q[-1,1]}}{\|P_n\|_{L_p[-1,1]}}$, where $\mathbb{P}_n$ denotes the set of algebraic polynomials of degree $\leq n$. It is known that, for any $0 < p, q \leq \infty$,

$$M_{q,p}(n,k) \asymp \begin{cases} n^{2k+2/p-2/q}, & \text{if } k > 2/q - 2/p, \\ n^k (\log n)^{1/q-1/p}, & \text{if } k = 2/q - 2/p, \\ n^k, & \text{if } k < 2/q - 2/p. \end{cases}$$

In 2009, T. Erdelyi found the order of $M_{q,p}^{(l)}(n,k)$ where the supremum is taken over the set of all absolutely monotone polynomials of order l (i.e., polynomials P_n such that $P_n^{(m)}(x) \geq 0$ for $0 \leq m \leq l$ and $x \in [-1,1]$). In particular, he showed that, for $1 \leq k \leq l/2$, $p \leq q$,

$$M_{q,p}^{(l)}(n,k) \asymp \left(n^2/l\right)^{k+1/p-1/q}.$$

This result implies that in case the $p \leq q$ the order of the Markov factor in constrained Makov-Nikolskii inequality is the same as in the classical one when l is fixed.

At the same time, A. Kroo and J. Szabados found the exact Markov factors for monotone polynomials of order k in L_∞ and L_1 norms.

In this talk, I will discuss sharp orders for all values $0 < p, q \leq \infty$. It turns out that Markov-Nikolskii type inequality can be significantly improved as long as $q < p$.

Keywords: polynomial inequalities, shape preserving approximation.
AMS Classification: 41A17.

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On the remainders of some asymptotic expansions

Stamatis Koumandos

Abstract

We study the remainders of several asymptotic expansions, such as, the logarithm of Euler's Gamma function, the logarithm of Barnes double and triple Gamma function.

We show that these remainders are completely monotonic functions of certain positive integer order on $(0, \infty)$. Furthermore, we show that these remainders are Laplace transforms of positive multiples of absolutely monotonic functions. We also establish sharp error estimates for the above mentioned asymptotic expansions.

This talk is based on joint work with Henrik L. Pedersen.

Keywords: asymptotic formula, Euler's Gamma function, Barnes double and triple Gamma function, completely monotonic functions, absolutely monotonic functions, error estimates.

AMS Classification: 33B15, 41A60, 41A80.

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Asymptotic formulae for generalized sampling operators*

Ilaria Mantellini and Carlo Bardaro

Abstract

An important class of discrete operators is given by the generalized sampling series introduced by P.L. Butzer and his school in Aachen (see [4, 5]), which have fundamental applications in signal processing, in particular in linear prediction by samples from the past of the signal to reconstruct. These operators are defined, in one-dimensional case, by

$$(G_n f)(x) = \sum_{k=-\infty}^{+\infty} \varphi(n(x - \frac{k}{n}))f(\frac{k}{n}), \quad n \in \mathbb{N}, \ x \in \mathbb{R},$$

where $\varphi : \mathbb{R} \to \mathbb{R}$ is a continuous function and the signal f belongs to suitable function spaces. Here we give an asymptotic expansion for G_n from which we deduce a Voronovskaja type formula, also in a quantitative form. Moreover, we consider a simple approach for the construction of linear combinations of generalized sampling operators which provide a better order of approximation. The use of linear combinations of positive linear operators was firstly introduced in a systematic form by P.L. Butzer in case of Bernstein polynomials and then widely developed by several authors for both integral or discrete operators (see e.g. the classical monograph by Ditzian and Totik "Moduli of smoothness").

Given $\alpha_1, \alpha_2, \ldots, \alpha_s \in \mathbb{R} \setminus \{0\}$ such that $\alpha_1 + \cdots + \alpha_s = 1$, we define the operator:

$$(G_n^s f)(x) = \sum_{i=1}^{s} \alpha_i (G_{in} f)(x),$$

and we look for coefficients α_i such that certain moments of higher order are null. In this way we obtain a linear system whose solution gives an operator with a high order of approximation. We apply this method to Bochner-Riesz and Jackson type kernels. We examine also combinations of translates of compactly supported central B-splines, which are of interest in the linear prediction theory.

Keywords: asymptotic formula, generalized sampling operators, linear combinations.
AMS Classification: 41A25, 41A60, 94A20.

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Remarks on L^p-convergence of Hermite interpolation

Giuseppe Mastroianni

Abstract

Necessary and sufficient conditions for the L^p-convergence of Lagrange interpolation on finite intervals are well-known. In this talk the previous conditions will be extended to the case of Hermite and Lagrange-Hermite interpolation based on Jacobi zeros. Error estimates for functions belonging to Sobolev or Zygmund spaces in weighted L^p-norm will be also given.

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Asymptotics and zeros of varying Laguerre-Sobolev type orthogonal polynomials*

Juan F. Mañas, Francisco Marcellán and Juan J. Moreno–Balcázar

Abstract

We consider the varying Sobolev inner product

$$(f,g)_n = \frac{1}{\Gamma(\alpha+1)} \int_0^\infty f(x)g(x)x^\alpha e^{-x}dx + M_n f^{(j)}(0)g^{(j)}(0), \quad \alpha > -1,$$

where $\{M_n\}_n$ is a sequence of nonnegative numbers satisfying

$$\lim_{n\to\infty} M_n n^\beta = M > 0, \quad \beta \in \mathbb{R}.$$

We obtain the Mehler–Heine type formulae for the orthogonal polynomials with respect to the above inner product which allows us to describe the asymptotic behavior of this family of polynomials on compact subsets of the complex plane. As a consequence, we deduce the asymptotics for the corresponding zeros. We remark that we tackle the general case and, in this way, we generalize some results obtained in [1] and [2], and also in [3] where the constant case ($M_n = M$, for all n) was treated.

Keywords: Mehler-Heine formulae, Laguerre orthogonal polynomials, zeros.
AMS Classification: 33C47, 42C05.

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Lagrange interpolation with exponential weights on $(-1, 1)$

Giuseppe Mastroianni and Incoronata Notarangelo

Abstract

We discuss the problem of approximating functions defined on $(-1, 1)$ with exponential growth for $|x| \to 1$ by means of polynomial in weighted function spaces. To this aim we consider two interpolation processes based on the zeros of orthonormal polynomials with respect to exponential weights. We will show convergence results and error estimates in weighted L^p and uniform metric. In particular, in some suitable function spaces, the related interpolating polynomials behave essentially like the polynomial of best approximation.

Keywords: orthogonal polynomials, Lagrange interpolation, approximation by polynomials, exponential weights.

AMS Classification: 41A05, 41A10.

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Turán type inequalities for the partial sums of the generating functions of Bernoulli and Euler numbers*

Henrik L. Pedersen

Abstract

Turán type inequalities for the partial sums of the generating functions of the Bernoulli and Euler numbers are established. They are shown to follow from a general result relating Turán inequalities of partial sums with Turán inequalities of the corresponding remainders in any Taylor expansion.

This is joint work with Stamatis Koumandos.

Keywords: Turan inequality, Bernoulli numbers, Euler numbers, generating functions, remainders.

AMS Classification: 11B68, 41A60, 41A80.

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Pi, Archimedes and circular splines

Paul Sablonnière

Abstract

In his famous treatise *On the quadrature of the circle*, Archimedes gave the following upper and lower bounds : $3+\frac{10}{71}<\pi<3+\frac{1}{7}$. For this purpose, he computed the *perimeters* of inscribed and circumscribed polygons to the unit circle, starting from the regular hexagon and doubling the number of edges up to the regular polygon with 96 edges. On the other hand, he also computed the *area of a sector of parabola* delimited by a secant and two tangents. From that result, he could have constructed the uniform piecewise parabolic circular curves which are tangent to the circle at the vertices or extreme points of parabolic arcs. Thus, he would have deduced approximations of π by computing the areas inside those closed curves. The latter are C^1 quadratic circular splines, and constitute a particular case of those described by Schoenberg (1971).
In China, the mathematician Liu Hui (third century), in his comments to the *Nine Chapters on the Mathematical Art*, had the idea of computing the *areas* of regular inscribed and circumscribed polygons. He got the approximation 3.14 by using a regular polygon of 96 sides. In the present paper, we compute approximate values of π deduced from the *areas* of inscribed and circumscribed quadratic and cubic circular splines to the unit circle. Similar results on circular splines of higher degrees can be obtained in the same way and will be published elsewhere. We also compare the latter results to those obtained via de Rham's (1956) and Merrien's (1992) subdivision schemes.

Keywords: exponential, circular splines, approximation of π.
AMS Classification: 41A60.

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Truncated Shannon sampling operators*

Gert Tamberg

Abstract

For the uniformly continuous and bounded functions $f \in C(\mathbb{R})$ the generalized sampling series are given by $(t \in \mathbb{R};\ W > 0)$

$$(S_W f)(t) := \sum_{k=-\infty}^{\infty} f(\frac{k}{W}) s(Wt - k), \tag{1}$$

where the condition for the operator $S_W : C(\mathbb{R}) \to C(\mathbb{R})$ to be well-defined is

$$\sum_{k=-\infty}^{\infty} |s(u-k)| < \infty \quad (u \in \mathbb{R}), \tag{2}$$

the absolute convergence being uniform on compact intervals of $\mathbb{R}$.

A systematic study of sampling operators (1) for arbitrary kernel functions s with (2) was initiated at RWTH Aachen by P. L. Butzer and his students since 1977 (see [3, 4] and references cited there).

We study (see [5, 6] and references cited there) an even band-limited kernel s, i.e. $s \in B^1_\pi$, defined by an even window function $\lambda \in C_{[-1,1]}$, $\lambda(0) = 1$, $\lambda(u) = 0$ $(|u| \geqslant 1)$ by the equality

$$s(t) := s(\lambda; t) := \int_0^1 \lambda(u) \cos(\pi t u)\, du. \tag{3}$$

In fact, this kernel is the Fourier transform of $\lambda \in L^1(\mathbb{R})$.

When applying (1) in practice, one must restrict oneself to finite sums. If the kernel s is with finite support, like B-spline kernels in [4], then we have a finite sum. If we use bandlimited kernels s, defined by (3), we must restrict ourself to finite sums which we shall take here to be symmetric about $\lfloor Wt \rfloor$ and make longer and longer to improve accuracy.

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The truncation error of classical WKS sampling operator is well-studied (see [1, 2]). In the following we give some estimates of truncation error for generalized sampling operators, defined with some bandlimited kernels in form (3), generalizing the results of [7].

In following we define the truncation error for sampling series (1). Let us consider the truncated version of the generalized Shannon sampling series (1)

$$(S_{W;N}f)(t) := \sum_{k=\lfloor Wt\rfloor-N+1}^{\lfloor Wt\rfloor+N} f\left(\frac{k}{W}\right) s(Wt-k).$$

The problem is how to estimate for $f \in C(\mathbb{R})$ the truncation error. In [7] we proved for some sampling operators the estimates of truncation error in the form

$$\|S_W f - S_{W;N} f\|_\infty \leqslant \|f\|_\infty C_s(N).$$

Now we prove the corresponding estimates of truncation errors in the form

$$\|S_W - S_{W;N}\|_{C\to C} = C_s(N).$$

Keywords: sampling operators, sampling series, approximation, truncation error.
AMS Classification: 41A28, 41A40, 41A60.

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Multivariate Sampling Kantorovich operators: Applications to digital images for Biomedical and Engineering problems*

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Abstract

We consider the multivariate Sampling Kantorovich operators of the form (see [3]):

$$(S_w^\chi f)(\underline{x}) \;=\; \sum_{\underline{k}\in\mathbb{Z}^n} \chi(w\underline{x}-t_{\underline{k}}) \left[\frac{w^n}{A_{\underline{k}}}\int_{R_{\underline{k}}^w} f(\underline{u})\, d\underline{u}\right] \qquad (\underline{x}\in\mathbb{R}^n,\ w>0), \qquad \text{(I)}$$

where $\chi : \mathbb{R}^n \to \mathbb{R}$ is a kernel function satisfying suitable assumptions, $t_{\underline{k}} = (t_{k_1}, ..., t_{k_n})$ is such that $(t_{k_i})_{k_i\in\mathbb{Z}}$, $i = 1, 2, ..., n$ are suitable sequences of real numbers. Moreover, $A_{\underline{k}} = \Delta_{k_1} \cdot \Delta_{k_2} \cdot ... \cdot \Delta_{k_n}$ with $\Delta_{k_i} = t_{k_i+1} - t_{k_i}$, $i = 1, 2, ..., n$,

$$R_{\underline{k}}^w \;=\; \left[\frac{t_{k_1}}{w}, \frac{t_{k_1+1}}{w}\right] \times \left[\frac{t_{k_2}}{w}, \frac{t_{k_2+1}}{w}\right] \times ... \times \left[\frac{t_{k_n}}{w}, \frac{t_{k_n+1}}{w}\right] \qquad (w>0),$$

and $f : \mathbb{R}^n \to \mathbb{R}$ is a locally integrable function for which the series (I) is convergent for every $\underline{x} \in \mathbb{R}^n$. We use the above operators to implement an algorithm for the treatement of digital images. In particular, we show that the developed theory in [3] allows us to reconstruct and to enhance the original images. Applications are given by using computer tomography images for the detection of deseases resulting from abdominal aortic aneurysms. Moreover, we apply the above theory and the related algorithm in order to process masonries images to study the texture thus separating the stones from the mortar, so facing an important problem in civil engineering. In order to deal with the last application, we also use thermographic images.

Keywords: multivariate sampling Kantorovich operators, Orlicz spaces, modular convergence, image processing, biomedical and engineering images.

AMS Classification: 41A35, 46E30, 47A58, 47B38, 94A12, 94A20, 97R30.

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