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Introduction

The fourth edition of the Jaen Conference on Approximation Theory continues the series of meetings at the city of Úbeda that started in 2000. Since then, this event has taken place every year steadily and always at Úbeda. On each occasion a group of outstanding mathematicians working in the field of the Approximation Theory met together to share their knowledge and new ideas. Therefore the Jaen and Ubeda conferences on Approximation have been for many years a forum where to develop scientific exchange and to start new research projects. A clear consequence of this all is the creation of the Jaen Journal on Approximation supported by many of the mathematicians that are our habitual collaborators in the meetings.

Moreover, the participants are offered the opportunity to strengthen the personal contact between the members of the Approximation community. Actually, to encourage this aspect, one of the main purposes of the organizers has always been, along with the scientific part, to programme a long list of recreational activities.

Although the current times are not the best, we are trying again to achieve the quality of the preceding meetings in all aspects. Visits to the historic locations, tasting the traditional gastronomy and activities for accompanying people are again present to try once more to make the meeting a window to the province of Jan and the culture of Andalucía and Spain.

The Organizing Committee

Scientific Program

	June, 24th-Monday	
9:30-10:30	OPENING CEREMONY (Úbeda Townhall)	
	SESSION 1 (Chairperson M. Jiménez-Pozo)	UNED (Úbeda)
10:30-11:30	Gradimir V. Milovanović (p. 10)	
11:30-12:00	**Coffee Break**	
	SESSION 2 (Chairperson G. Mastroianni)	
12:00-13:00	José A. Adell (p. 3)	
13:00-13:30	Incoronata Notarangelo (p. 22)	
14:00-	**Lunch** (Restaurante el Marqués)	
20:30-	**Dinner** (Parador de Úbeda)	
22:30-	**Visit to Úbeda**	

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	SESSION 3 (Chairperson A. Kroó)	UNED (Úbeda)
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10:30-11:00	Georg Zimmermann (p. 28)	
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	SESSION 4 (Chairperson G. Milovanovic)	
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12:30-13:00	Vasiliy Prokhorov (p. 24)	
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	June, 26th-Wednesday	
	SESSION 5 (Chairperson J.M. Peña)	UNED (Úbeda)
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10:30-11:00	José María Almira Picazo (p. 15)	
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11:30-11:45	**POSTER SESSION**	
	SESSION 6 (Chairperson J. A. Adell)	
11:45-12:45	Miguel Antonio Jiménez-Pozo (p. 6)	
12:45-13:15	Samuel G. Moreno (p. 19)	
14:00-	**Lunch** (Restaurante el Marqués)	
21:30-	**Gala Dinner** (Honrados Viejos del Salvador)	

	June, 27th-Thursday	
	SESSION 7 (Chairperson M. Buhmann)	UNED (Úbeda)
9:30-10:30	Charles A. Micchelli (p. 9)	
10:30-11:00	E. Corbacho (p. 17)	
11:00-11:30	**Coffee Break**	
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Poster Session

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- Rodolfo Toledo (p. 26).

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Invited Lectures

Short Talks/Posters

Invited Lectures

Differential calculus for linear operators and its applications to analytical number theory*

José A. Adell

Abstract

In the first part of the talk, we develop a differential calculus for linear operators represented by a family of finite signed measures, in particular, by stochastic processes. Such a calculus is based on the notions of g-derived operators and processes and g-integrating measures, g being a right-continuous nondecreasing function. Depending on the choice of g, this differential calculus works for non-smooth functions and under weak integrability conditions. For linear operators represented by stochastic processes, we provide a characterization criterion of g-differentiability in terms of characteristic functions of the random variables involved. Various illustrative examples, involving the binomial, the negative binomial and the gamma processes, are considered.

The second part of the talk is concerned with applications to analytical number theory, mainly referring to fast computations, by means of series with a geometric rate of convergence, of well known functions and constants, such as the logarithm, the inverse tangent, the Catalan constant, and the alternating zeta function, among others. Special attention is devoted to the computation of the sequence $(\gamma_n(a))_{n\geq 0}$ of the Stieltjes constants appearing in the Laurent expansion of the Hurwitz and Riemann zeta functions. We provide different approximations of each constant $\gamma_n(a)$ by means of finite sums or series involving Bernoulli numbers. The particular case of the Euler-Mascheroni constant is discussed in more detail.

Keywords: differential calculus, gamma process, negative binomial process, Catalan constant, Riemann zeta function, Stieltjes constants, Euler-Mascheroni constant.

AMS Classification: 26A48, 33B15, 60E05.

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*This work has been partially supported by research grants MTM2011-23998, DGA E-64, and by FEDER funds.

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Multiquadrics and its Approximation properties: Recent results

Martin Buhmann

Abstract

In a recent paper with F. Dai we establish new convergence results for quasi-interpolation and other approximations using in particular multiquadrics radial basis functions, but also larger classes of radial functions. Several in particular pointwise results are given, also in general, weighted norms and also for derivatives. Some of them improve the convergence orders known before. We consider approximations by compression for example as well.

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Hölder classes of functions and polynomial approximation

Miguel Antonio Jiménez-Pozo

Abstract

In this talk we present different classes of Hölder spaces in which the approximation by polynomials (or by functions near to polynomials) is considered. We review several classical results before the introduction of different classes of Hölder spaces. These classes include functions of infinitely many variables, weighted integral functions, sign sensitive weighted integral functions, Besov classes, abstract spaces and others. Polynomial approximation is possible in most of these spaces and the degree of approximation and inverse results in different classes are given. Results usually refer to small Hölder or Lipschitz classes but with the same parameter α in the definition of Hölder spaces and not with respect to a $\beta < \alpha$. The talk includes historical remarks, related papers, recent results and open problems.

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Bernstein-Markov type inequalities on star–like domains in $\mathbb{R}^d$ with application to norming sets

András Kroó

Abstract

Bernstein- Markov type inequalities widely used in various areas of analysis are related to estimating derivatives of polynomials on various domains. In this talk we shall give a survey of the history of this problem and present some new Bernstein-Markov type inequalities for multivariate polynomials on star like domains.

These new inequalities will be shown to be useful in the study of cardinality of norming sets, or admissible meshes. Admissible meshes are applied in various areas, for instance they are used for discrete least squares approximation, extracting discrete extremal sets optimal for interpolation, scattered data interpolation, cubature formulas, etc. Admissible meshes with asymptotically minimal number of points, called optimal meshes, are particularly important for applications. We shall review recent results on existence of optimal meshes in star like domains.

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Polynomial approximation on the real semiaxis

G. Mastroianni and I. Notarangelo

Abstract

The talk is a brief survey on polynomial approximation of functions, defined on $\mathbb{R}^+$ and exponentially monotonic at the endpoints of the semiaxis.

We introduce new function spaces, suitable moduli of smoothness and equivalent $K-$functionals. With these moduli of smoothness, we prove the Jackson and Stechkin inequalities.

Moreover, we show polynomial inequalities and some results concerning Lagrange interpolation.

Keywords: Weighted polynomial approximation, exponential weights, unbounded interval, Jackson theorem, Stechkin inequality, Lagrange interpolation.

AMS Classification: 41A10, 41A25, 41A05.

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Polynomial Approximation to an ideal filter

Charles A. Micchelli

Abstract

In this talk we shall discuss some recent work on optimal polynomial filter design obtained jointly with Yi Wang of Auburn University, Alabama and Jianzhong Wang from San Houston State University, Texas.

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Methods for computation of slowly convergent series and finite sums basing on Gauss-Christoffel quadratures*

Gradimir V. Milovanović

Abstract

For slowly convergent series and finite sums, which are appeared in many problems in mathematics, physics and other sciences, there are several numerical methods based on linear and nonlinear transformations. In general, starting from the sequence of partial sums of the series, these transformations give other sequences with a faster convergence to the sum of the series. There is a rich literature on this subject (cf. references in Mastroianni and Milovanović [3]).

In this lecture we give an account on some summation processes for series ($n = \infty$) and finite sums,

$$\sum_{k=1}^{n} (\pm 1)^k f(k), \tag{1}$$

with a given function $z \mapsto f(z)$ with certain properties with respect to variable z, based on ideas related to Gauss-Christoffel quadratures. In a general case, the function f can depend on other parameters, e.g., $f(z; x, \ldots)$, so that these summation proceses can be applied also to some classes of functional series, not only to numerical series.

The basic idea in our methods is to transform the sum (1) to an integral with respect to some measure $\mathrm{d}\mu$ on $\mathbb{R}_+$, and then to approximate this integral by a finite quadrature sum,

$$\sum_{k=1}^{n} (\pm 1)^k f(k) = \int_{\mathbb{R}_+} g(t)\,\mathrm{d}\mu(t) \approx \sum_{\nu=1}^{N} A_\nu g(x_\nu), \tag{2}$$

where the function g is connected with f in some way, and the weights $A_\nu \equiv A_\nu^{(n)}$ and abscissae $x_\nu \equiv x_\nu^{(n)}$, $\nu = 1, \ldots, N$, are chosen in such a way as to approximate closely the sum (1) for a large class of functions with a relatively small number $N \ll n$. In our approach we take the Gaussian quadrature sums as the sum on the right-hand side in (2).

*This work was supported in part by the Serbian Ministry of Education, Science and Technological Development.

In construction of Gaussian quadratures with respect to the nonstandard measures $d\mu$ on $\mathbb{R}_+$ we use the recent progress in symbolic computation and variable-precision arithmetic and our MATHEMATICA package `OrthogonalPolynomials` [1], [6]. The approach enables us to overcome the numerical instability in generating coefficients of the three-term recurrence relation for the corresponding orthogonal polynomials with respect to the measure $d\mu$ on $\mathbb{R}_+$.

Keywords: Summation, Gaussian quadratures, weight function, three-term recurence relation, convergence, contour integration.

AMS Classification: 40A25, 65D32, 30E20, 33C52, 33C90.

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Accuracy and Stability: recent advances in C.A.G.D.*

J.M. Peña and J. Delgado

Abstract

Recent results on accurate and stable computational methods for Approximation Theory and Computer Aided Geometric Design are discussed. The relationship with numerical methods for some structured classes of matrices related to positivity is analyzed. In fact, for these classes of matrices many computations can be performed with high relative accuracy. Related problems are considered. For instance, we consider the progressive iteration approximation and stable evaluation algorithms for rational surfaces as well as alternative models to the rational model.

Keywords: accurate computations, stability, shape preserving representations, progressive iteration, evaluation algorithms.

AMS Classification: 65G50, 65D17, 41A20, 65F05.

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*This work has been partially supported by the Spanish Research Grant MTM2012-31544, Gobierno de Aragón and Fondo Social Europeo.

Short Talks/Posters

On subspaces of distributions which are Δ^m-invariant

José María Almira Picazo

Abstract

We study the finite dimensional spaces V which are invariant under the action of the finite differences operator Δ_h^m. Concretely, we prove that if V is such an space, there exists a finite dimensional translation invariant space W such that $V \subseteq W$. In particular, all elements of V are exponential polynomials. Furthermore, V admits a decomposition $V = P \oplus E$ with P a space of polynomials and E a translation invariant space. As a consequence of this study, we prove a generalization of a famous result by P. Montel which states that, if $f : \mathbb{R} \to \mathbb{C}$ is a continuous function satisfying $\Delta_{h_1}^m f(t) = \Delta_{h_2}^m f(t) = 0$ for all $t \in \mathbb{R}$ and certain $h_1, h_2 \in \mathbb{R} \setminus \{0\}$ such that $h_1/h_2 \notin \mathbb{Q}$, then $f(t) = a_0 + a_1 t + \cdots + a_{m-1} t^{m-1}$ for all $t \in \mathbb{R}$ and certain complex numbers $a_0, a_1, \cdots, a_{m-1}$. We demonstrate, with quite different arguments, the same result not only for ordinary functions $f(t)$ but also for complex valued distributions. Finally, we also consider in this paper the subspaces V which are $\Delta_{h_1 h_2 \cdots h_m}$-invariant for all $h_1, \cdots, h_m \in \mathbb{R}$. This talk is based on the recently accepted paper [1].

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Approximation with the RAFU method

E. Corbacho

Abstract

The **RAFU** (radical functions) method is an original and unknown approximation procedure we can use in Approximation Theory. This method becomes uniform in the set of continuous functions defined in a closed real interval. The uniform stability of this approximation procedure by radical functions improves the instability of the interpolation by means of polynomials. The degree of uniform approximation with the **RAFU** method is better than the degree of uniform approximation by arbitrary polynomials and Bernstein polynomials for continuous functions highly irregular. In this work we will show we can employ the mentioned approximation procedure to approach functions not necessarily continuous and functions defined in all real line.

Keywords: RAFU method, approximation theory, radical functions, approximation algorithm.

AMS Classification: 41A30, 37L65.

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Orthogonality of the Meixner-Pollaczek polynomials beyond Favard's theorem*

Samuel G. Moreno and Esther M. García–Caballero

Abstract

In this talk we extend the family of Meixner-Pollaczek polynomials $\{P_n^{(\lambda)}(\cdot;\phi)\}_{n=0}^{\infty}$, classically defined for $\lambda > 0$ and $0 < \phi < \pi$, to arbitrary complex values of the parameter λ, in such a way that both polynomial systems (the classical and the new *generalized* ones) share the same three term recurrence relation. The values $\lambda_N = (1 - N)/2$, with N a positive integer, are the only ones for which no orthogonality condition can be deduced from Favard's theorem. We introduce a non-standard discrete-continuous inner product with respect to which the generalized Meixner-Pollaczek polynomials $\{P_n^{(\lambda_N)}(\cdot;\phi)\}_{n=0}^{\infty}$ become orthogonal.

The talk summarizes the recently published results in [16].

Keywords: Meixner-Pollaczek polynomials, hypergeometric orthogonal polynomials, Favard's theorem, non-standard inner product.

MSC 2010: 33C45, 42C05.

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Gaussian quadrature rules with an exponential weight on the real semiaxis

G. Mastroianni, G. V. Milovanović and I. Notarangelo

Abstract

We consider Gaussian rules related to the weight function

$$w(x) = \mathrm{e}^{-x^{-\alpha}-x^{\beta}}, \quad \alpha > 0,\ \beta > 1, \quad x \in (0, +\infty).$$

We first study the behavior of the "ordinary" Gaussian rule in spaces of continuous functions with weighted uniform metric, proving that, under proper assumptions, the formula converges with the order of the best weighted polynomial approximation and with geometric rate for infinitely differentiable functions. Nevertheless the error of this formula does not converge with the optimal rate in weighted L^1-Sobolev spaces.

To overcome this problem, we suggest a "truncated" Gaussian rule. This formula converges with the same order of the ordinary Gaussian rule for continuous functions. Moreover, it converges with the order of the best polynomial approximation for functions belonging to weighted L^1-Sobolev.

Finally, some numerical tests are shown.

Keywords: Gaussian quadrature rules, orthogonal polynomials, weighted polynomial approximation, exponential weights, unbounded intervals.

AMS Classification: 41A10, 65D30, 65D32.

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On approximation by rational functions

Vasiliy Prokhorov

Abstract

Let f be a function analytic at the point $z = 0$. We represent the function f in some neighborhood of the point $z = 0$ by a convergent power series

$$f(z) = \sum_{k=0}^{\infty} f_k z^k.$$

One of the classical constructive methods of approximation of analytic functions given by a power series is the Padé approximation (see, for example, the monograph [3]). This method is a method of rational approximation with free poles (there is no restriction on poles). Padé approximants are locally the best rational approximants to a given power series. These approximants localize the singular points of a function determined by the power series and enable us to obtain an efficient, and under certain conditions, analytic continuation of the power series beyond its circle of convergence. In this presentation we consider a constructive method of approximation of analytic functions given by a power series based on ideas of the theory of Hankel operators. As Padé approximants, these approximants are rational functions and constructed in terms of the coefficients f_k. We discuss some properties of the approximants and the corresponding Hankel operators.

Keywords: rational approximation, meromorphic approximation, Padé approximation, Hankel operators.
AMS Classification: 41A20, 41A21, 30E10, 47B35.

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Representative product systems on the complete product of quaternion groups*

Rodolfo Toledo

Abstract

A modern point of view in Fourier analysis is to consider orthonormal systems defined on locally compact groups. Therefore, the study of Walsh series should be performed by representing the Walsh functions as the characters of the dyadic group, i.e., the complete product of the discrete cyclic group of order 2 with the product of topologies and measures. Vilenkin in 1947 generalized this structure studying the complete product of arbitrary cyclic groups. In [1] the authors generalized the Vilenkin systems. The main idea is to take the complete direct product of arbitrary finite groups, even non-abelian groups. The characters of a finite non-abelian group can not form a complete system because their number is less than the order of the group. The missing functions can be obtained by computing the representations of the group. Product systems formed by characters and normalized coordinate functions of these representations are called representative product systems.

In Fourier analysis several properties and results differ considerably if they are defined on non-abelian groups. For instance, Walsh and Vilenkin-systems have only functions with module 1, but a representative product system on the complete product of finite non-abelian groups is not necessarily uniformly bounded and it takes the value 0. On the other hand, we also obtain some negative results concerning convergence in L^p-norm ($1 < p < \infty$) of Fourier series and and Cesàro means for certain representative product systems (see [2]).

In this study I deal with specific representative product systems defined on the complete product of quaternion groups. We show the fact that some properties of these systems are very similar to the properties of Walsh and Vilenkin-systems and in these cases we obtain positive results with respect to convergence in L^p-norm ($1 < p < \infty$) of Fourier series and and Cesàro means.

Keywords: Fourier analysis, Walsh system, Vilenkin system, representative product systems, complete product of quaternion groups, convergence in norm, Cesàro means.

AMS Classification: 42C10.

*Research supported by project TÁMOP-4.2.2.A-11/1/KONV-2012-0051.

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A proof for the Gasca-Maeztu Conjecture for $n = 5$*

Hakop Hakopian, Kurt Jetter and Georg Zimmermann

Abstract

An n-poised set in two dimensions is a set of nodes admitting unique bivariate interpolation with polynomials of total degree at most n. We are interested in poised sets with the property that all fundamental polynomials are products of linear factors. In 1982, M. Gasca and J. I. Maeztu conjectured that every such set necessarily contains $n+1$ collinear points. Up to now, this had been confirmed only for $n \leq 4$, the case $n = 4$ having been proved for the first time in 1990 by J. R. Busch [1], later with different methods by J. M. Carnicer and M. Gasca [2], and later again with different methods by the authors [3]. We prove the case $n = 5$ with new methods that might also be useful in deciding the still open cases for $n \geq 6$.

Keywords: polynomial interpolation, Gasca-Maeztu conjecture, fundamental polynomial, maximal line, poised.

AMS Classification: 41A05, 41A63, 14H50.

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*Support by the DAAD grant # A/12/05111, and State Committee of Science of Armenia, grant # 11-1A290, is gratefully acknowledged.

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