V Jaen Conference on Approximation Theory

Úbeda, Jaén, Spain, June 23rd-26th, 2014

Approximation Theory, Computer Aided Geometric Design, Numerical Methods and Applications

Scientific Committee

- András Kroó
  Alfréd Rènyi Institute of Mathematics,
  Hungarian Academy of Sciences, Hungary
- Charles A. Micchelli
  University at Albany,
  State University of New York, USA
- Dany Leviatan
  University of Tel-Aviv, Israel
- Doron S. Lubinsky
  School of Mathematics,
  Georgia Institute of Technology, USA
- Francesco Altomare
  Departimento di Matematica,
  Facoltà di Scienze,
  Università degli Studi di Bari, Italy
- Francisco Marcellán,
  Universidad Carlos III de Madrid, Spain
- Francisco-Javier Muñoz-Delgado
  Universidad de Jaén, Spain
- Giuseppe Mastroianni
  Università degli Studi della Basilicata,
  Italy
- Hrushikesh N Mhaskar
  California State University, USA
- József Szabados
  Alfréd Rényi Institute of Mathematics,
  Hungarian Academy of Sciences,
  Hungary
- Igor Shevchuk
  University of Kyiv, Ukraine
- José Antonio Adell
  Universidad de Zaragoza, Spain
- Kirill A. Kopotun
  University of Manitoba, Canada
- Kurt Jetter
  Universität Hohenheim, Germany
- Manfred v. Golitschek
  Universität Würzburg, Germany
- Mariano Gasca
  Universidad de Zaragoza, Spain
- Marie-Laurence Mazure
  Universitè Joseph Fourier de
  Grenoble, France
- Martin Buhmann
  Justus-Liebig-Universität Giessen,
  Germany
- Miguel A. Jiménez-Pozo
  Universidad de Puebla, México
- Paul Sablonnière
  INSA Rennes, France
- Tom Lyche
  University of Oslo, Norway
- Vilmos Totik
  University of South Florida,
  Tampa, USA &
  University of Szeged, Hungary
- Zeev Ditzian
  University of Alberta, Canada

Scientific Secretary

- Daniel Cárdenas Morales
  Universidad de Jaén,
  Jaén, Spain
- Antonio Jesús López Moreno
  Universidad de Jaén,
  Jaén, Spain

Plenary Speakers

- Alicia Cachafeiro,
  Universidad de Vigo,
  Spain
- Erik Koelink,
  Radboud University Nijmegen,
  The Netherlands
- Paul Nevai,
  KAU, Jeddah, Saudi Arabia &
  Upper Arlington (Columbus), Ohio, USA
- Jungho Yoon,
  Ewha W. University,
  South Korea,
- Kurt Jetter,
  Universität Hohenheim,
  Germany
- Guillermo López Lagomasino,
  Universidad Carlos III de Madrid,
  Spain
- Paul Sablonnière,
  INSA de Rennes,
  France

Organizing Committee

- Francisco Javier Muñoz Delgado (*Chairman*)
- Daniel Cárdenas Morales
- Antonio Damas Serrano
- Esther García Caballero
- Miguel Ángel García Muñoz
- Samuel Gómez Moreno
- Pedro Garrancho García
- Joaquín Jódar Reyes
- Antonio Jesús López Moreno
- María Francisca Molina Alba
- Juan Martínez Moreno
- Juan Navas Ureña
- José María Quesada Teruel
- Juan Francisco Ruiz Ruiz

Department of Mathematics,
University of Jaén, Spain

Introduction

This Conference is an activity of the Jaen Approximation Project. Jaen Approximation Project has organized ten editions of the Ubeda Meeting on Approximation and five editions of the Jaen Conference on Approximation. It also issues the Jaen Journal on Approximation since 2009.

The objective of these conferences is to provide a useful and nice forum for researchers in the subjects to meet and discuss. In this sense, the conference program has been designed to keep joined the group during four days with a program full of scientific and social activities.

The Conference will be devoted to some significant aspects on Approximation Theory, Computer Aided Geometric Design, Numerical Methods and the Applications of these fields in other areas.

It features seven invited speakers (Alicia Cachafeiro Cachafeiro, Kurt Jetter, Erik Koelink, Guillermo Tomás López Lagomasino, Paul Nevai, Paul Sablonnière, Yungho Yoon) who will give 50 minutes plenary lectures. Researchers were invited to contribute with a talk or a poster. We have scheduled 30 talks and a poster session.

We also provided the possibility to organizing mini-symposia on a subject of current interest. The following proposal has been accepted: "Orthogonal Polynomials in Approximation Theory" organized by Juan José Moreno-Balcázar, Teresa E. Pérez and Miguel Piñar.

This Conference is especially dedicated to these hundreds of people from more than 40 countries all over the world that have supported the journal, as editors, authors, referees and subscribers. The Conference is held in Úbeda, what gives participants the opportunity to visit World Heritage Sites and taste a wide culinary variety.

We hope that you all enjoy the Conference, both participants and accompanying people. We are grateful to all those who have made this project a reality; the University of Jaén (Vicerrectorado de Investigación and Departamento de Matemáticas), Diputación Provincial de Jaén, Ayuntamiento de Úbeda and UNED. Here we emphasize our commitment to keep on working to improve our university and our province.

The Organizing Committee

Scientific Program

	June, 22nd-Sunday
21:00-	**Dinner** (Restaurante El Marqués)

	June, 23rd-Monday	
9:00- 9:30	**Registration**	UNED (Úbeda)
9:30- 9:50	**OPENING CEREMONY**	
	SESSION 1 (Chairperson D. Leviatan)	
9:50-10:40	Paul Nevai (p. 10)	
10:40-11:00	József Szabados (p. 83)	
11:00-11:15	**Coffee Break**	
	SESSION 2 (Chairperson A. Kroo)	
11:15-11:35	José A. Adell (p. 17)	
11:35-11:55	Didem Aydın Arı(p. 29)	
11:55-12:15	Özge Dalmanoğlu (p. 40)	
12:15-12:35	Béla Nagy (p. 63)	
12:35-12:55	J. Marzo (p. 57)	
12:55-13:20	**POSTER SESSION 1**	
	SESSION 3 (Chairperson J. A. Adell)	
13:20-13:40	Giuseppe Mastroianni (p. 59)	
13:40-14:00	Incoronata Notarangelo (p. 65)	
14:00-14:20	Rachid Ait-Haddou (p. 19)	
14:30-	**Lunch** (Restaurante El Marqués)	
19:00-21:00	**Visit to Úbeda**	
21:00-	**Olive oil tasting and dinner** (Parador de Úbeda)	

	June, 24th-Tuesday	
	SESSION 4 (Chairperson M. Buhmann)	UNED (Úbeda)
9:30-10:20	Kurt Jetter (p. 5)	
10:20-10:40	Elena E. Berdysheva (p. 32)	
10:40-11:00	Georg Zimmermann (p. 86)	
11:00-11:15	**Coffee Break**	
	SESSION 5 (Chairperson P. Sablonnière)	
11:15-11:35	Hrushikesh Mhaskar (p. 62)	
11:35-11:55	Hermann Render (p. 77)	
11:55-12:15	Tivadar Danka (p. 42)	
12:15-12:35	Scott N. Kersey (p. 53)	
12:35-12:55	Sergei Kalmykov (p. 51)	
12:55-13:10	**BREAK**	
	SESSION 6 (Chairperson H. N. Mhaskar)	
13:10-14:00	G. López Lagomasino (p. 9)	
14:00-14:20	Vitaly E. Maiorov (p. 54)	
14:30-	**Lunch** (Restaurante El Marqués)	
18:00-21:30	**Visit to Baños de la Encina castle and Baeza**	
21:30-	**Dinner** (Jabalquinto Palace - Baeza)	

	June, 25th-Wednesday	
9:00- 9:45	**Guided tour of the Synagogue of the Water**	
9:45-10:15	**Summer solstice**	
	SESSION 7 (Chairperson M. L. Mazure)	UNED (Úbeda)
10:30-11:20	Jungho Yoon (p. 12)	
11:20-11:35	**Coffee Break**	
	MINI-SYMPOSIUM (session 1): Orthogonal Polynomials in Approximation Theory (Chairperson G. López Lagomasino)	
11:35-11:55	J. Petronilho (p. 71)	
11:55-12:15	M. Alfaro (p. 22)	
12:15-12:35	José Luis Cardoso (p. 36)	
12:35-12:55	I. Cação (p. 34)	
12:55-13:15	Ruymán Cruz Barroso (p. 38)	
13:15-13:30	**BREAK**	
	SESSION 8 (Chairperson G. Mastroianni)	
13:30-14:20	Alicia Cachafeiro (p. 3)	
14:20-14:40	Marie-Laurence Mazure (p. 60)	
15:00-	**Lunch** (Parador de Úbeda)	
18:00-	**Jaen J. Approx. editorial board meeting**	
21:30-	**Gala dinner** (Hospital de Santiago)	

	June, 26th-Thursday	
	SESSION 9 (Chairperson K. Jetter)	UNED (Úbeda)
9:30-10:20	Paul Sablonnière (p. 11)	
10:20-10:40	J. M. Almira (p. 23)	
10:40-11:00	Á. P. Horváth (p. 47)	
11:00-11:15	**Coffee Break**	
	MINI-SYMPOSIUM (session 2): Orthogonal Polynomials in Approximation Theory (Chairperson A. Cachafeiro)	
11:15-11:35	Juan F. Mañas-Mañas (p. 55)	
11:35-11:55	Maria das Neves Rebocho (p. 75)	
11:55-12:15	Darío Ramos López (p. 73)	
12:15-12:35	J.F. Sánchez-Lara (p. 80)	
12:35-12:55	Teresa E. Pérez (p. 69)	
12:55-13:20	**POSTER SESSION 2**	
	SESSION 10 (Chairperson J. Szabados)	
13:20-14:10	Erik Koelink (p. 7)	
14:10-	**CLOSING CEREMONY**	
14:30-	**Lunch** (Restaurante El Marqués)	
19:00-21:00	**Visit to Sabiote**	
21:00-	**Dinner** (Sabiote Castle)	

Mini-symposium

Miny-symposium 'Orthogonal Polynomials in Approximation Theory'
(Organized by Juan José Moreno Balcázar, Teresa E. Pérez and Miguel Piñar)

Session 1: 25th-Wednesday, 11:35–13:15.

Session 2: 26th-Thursday, 11:15–12:55.

Poster Session

Session 1: 23rd-Monday, 12:55–13:20,

Session 2: 26th-Thursday, 12:55–13:20.

Index

Invited Lectures

Short Talks/Posters

Invited Lectures

Some topics in Hermite interpolation on the unit circle*

Alicia Cachafeiro

Abstract

The aim of this talk is to present some results and ideas related to Hermite interpolation on the unit circle with equally spaced nodal systems. The main topics covered in this overview are the obtention of explicit expressions for the interpolation polynomials, the study of the rate of convergence of the Hermite-Fejér interpolants and other related topics such as a Brutman type theorem.

Keywords: Hermite interpolation, barycentric expressions, rate of convergence.
AMS Classification: 41A05, 65D05, 42A15, 33E20.

Bibliography

[1] **E. Berriochoa, A. Cachafeiro, J. Díaz, J. R. Illán**, Explicit formulas for Hermite type interpolation on the circle and applications (submitted).

[2] **E. Berriochoa, A. Cachafeiro, J. Díaz, E. Martínez**, Rate of convergence of Hermite-Fejér interpolation on the unit circle, *J. Appl. Math.* **2013** (2013), Article ID 407128.

[3] **R. Bojaníc, F. H. Chêng**, Rate of Convergence of Hermite-Fejér polynomials for functions with derivartives of bounded variation, *Acta Math. Hung.* **59** (1-2) (1992) 91–102.

[4] **L. Brutman, I. Gopengauz, P. Vértesi**, On the Domain of Divergence of Hermite-Fejér Interpolating Polynomials, *J. Approx. Theory* **106** (2000) 287–290.

[5] **L. Daruis, P. González-Vera**, A note on Hermite-Fejér interpolation for the unit circle, *Appl. Math. Letters* **14** (2001) 997–1003.

[6] **A. Poghosyan**, Fourier formulae for equidistant Hermite trigonometric interpolation, *Asian-European Journal of Mathematics* **4** (1) (2011) 127–144.

*The research was supported by Ministerio de Ciencia e Innovación under grant number MTM2011-22713.

[7] **J. Szabados**, Optimal order of convergence of Hermite-Fejér interpolation for general systems of nodes, *Acta Sci. Math. (Szeged)* **57**(1-4) (1993) 463–470.

[8] **L. N. Trefethen**, Approximation theory and approximation practice, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2013.

Alicia Cachafeiro,
Departamento de Matemática Aplicada I,
Escuela de Ingeniería Industrial,
Universidad de Vigo,
36310 Vigo, Spain.
`acachafe@uvigo.es`

The Gasca-Maeztu conjecture reconsidered

Kurt Jetter

Abstract

The Gasca-Maeztu conjecture deals with bivariate polynomial interpolation on an n-poised set $X \subset \mathbb{R}^2$ of nodes, assuming that each fundamental polynomial factors completely into n linear factors (or lines). It states that, then, at least $n+1$ nodes from X must be collinear. Surprisingly, although the conjecture now stands for more than 30 years, c.f. [2], it has been verified up to now only for $n \leq 5$.

While the conjecture itself can be formulated as a combinatorial problem refering to a special geometry of incidence structures, the proofs for the non-trivial cases $n = 4$ and $n = 5$ also involve results from Algebraic Geometry. The talk will aim at giving an introduction to some of these methods.

There are various equivalent forms of the conjecture, if we ask the conjecture to hold for all $k \leq n$, for given $n \in \mathbb{N}$. The most striking ones are the existence of three non-concurrent lines containing $n+1$ nodes each and $3n$ nodes altogether, a result due to Carnicer and Gasca [1], and the statement that for each node $A \in X$, the remaining nodes are distributed in a Berzolari-Radon type way. This follows from our treatment in [3] of characterizing the geometry of interpolation lattices – satisfying the assumptions in the Gasca-Maeztu conjecture – by so-called maximal distribution sequences: the latter must be of type $(n+1, n, \ldots, 2)$, for each $A \in X$. Both these statements give the hope that, eventually, an inductive proof might be available.

Crucial in our proof of the case $n = 5$ is the close relation of the number of nodes on a line, and the number of uses of that line as a factor in the fundamental polynomials. Although some general results are available, they are still much too weak in order to have the hope of a quick answer to the problem.

The talk is based on joint work with Hakop Hakopian and Georg Zimmermann.

Keywords: bivariate polynomial interpolation, geometric condition, Chung-Yao natural lattice, Berzolari-Radon type set, maximal line, maximal distribution sequence.

AMS Classification: 41A05 41A63, 14H50.

Bibliography

[1] **J.M.Carnicer and M.Gasca**, On Chung and Yao's geometric characterization for bivariate polynomial interpolation. In: T.Lyche, M.-L.Mazure and L.L.Schumaker (eds.), *Curve and Surface Design*, St. Malo 2002, pp. 21–30, Nashboro Press, Brentwood (2003).

[2] **M.Gasca and J.I.Maeztu**, On Lagrange and Hermite interpolation in $\mathbb{R}^k$, *Numer. Math.* **39** (1982), 1–14.

[3] **H.Hakopian, K.Jetter and G.Zimmermann**, The Gasca-Maeztu conjecture for $n = 5$, *Numer. Math.* (2013), DOI 10.1007/s00211-013-0599-4.

Kurt Jetter,
Institut für Angewandte Mathematik unf Statistik,
Universität Hohenheim.
Kurt.Jetter@uni-hohenheim.de

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

Matrix-valued orthogonal polynomials

Erik Koelink, Ana M. de los Ríos and Pablo Román

Abstract

Matrix-valued orthogonal polynomials go back to the pioneering work of M.G. Krein around 1950. These are polynomials taking their values in the algebra of $N \times N$-matrices satisfying a suitable matrix-valued orthogonality relation. Many properties of matrix-valued orthogonal polynomials are along the lines of corresponding well-known properties of the scalar-valued orthogonal polynomials. Classically, the orthogonal polynomials of Jacobi, Laguerre and Hermite are eigenfunctions to a second order differential equation, and properties of these polynomials can be obtained from this result. For the matrix-valued case it is (in general) rather difficult to have well-understood families of matrix-valued orthogonal polynomials for arbitrary size N. In this lecture we concentrate on an explicit family of matrix-valued orthogonal polynomials for arbitrary size which can be considered as a matrix-valued analogue of Gegenbauer (or ultraspherical) polynomials. In particular, we discuss these matrix-valued polynomials from the perspective of eigenfunctions of two matrix-valued differential operators.

Keywords: matrix-valued orthogonal polynomials, Gegenbauer polynomials.
AMS Classification: 33C45, 34L10, 35C11.

Bibliography

[1] **E. Koelink, A.M. de los Ríos and P. Román**, Matrix-valued Gegenbauer polynomials, `arXiv:1403.2938`.

Erik Koelink,
IMAPP,
Radboud Universiteit Nijmegen, the Netherlands.
`e.koelink@math.ru.nl`

Ana M. de los Ríos,
Departamento de Análisis Matemático,

Universidad de Sevilla, Spain.
amdelosrios@us.es

Pablo Román,
CIEM, FaMAF,
Universidad Nacional de Córdoba, Argentina.
roman@famaf.unc.edu.ar

Convergence of type I Hermite-Padé approximants of systems of meromorphic functions*

G. López Lagomasino and S. Medina Peralta

Abstract

We present new results on the convergence of diagonal sequences of type I Hermite-Padé approximation for systems of meromorphic functions obtained through a vector rational modification with real coefficients of a Nikishisn system of functions. We show that the approximants not only recover the functions but also locate its poles taking account of their order.

Keywords: Hermite-Padé approximation, multiple orthogonal polynomials.
AMS Classification: 30E10, 41A21, 42C05.

Bibliography

[1] **A. A. Gonchar**, On convergence of Padé approximants for some classes of meromorphic functions, *Math. USSR Sb.* **26** (1975) 555–575.

[2] **E. A. Rakhmanov**, The Characterization of Generalized Convex Functions, *Math. USSR Sb.* **33** (1977) 243–260.

[3] **J. Bustamante and G. López Lagomasino**, Hermite-Padé approximation for Nikishin systems of analytic functions, *Math. USSR Sb.* **77** (1994) 367–384.

G. López Lagomasino,
Departamento de Matemáticas,
Universidad Carlos III de Madrid.
lago@math.uc3m.es

S. Medina Peralta,
Departamento de Matemáticas,
Universidad Carlos III de Madrid.
smedina@math.uc3m.es

*MICINN, grant MTM 2012-36732-C03-01.

The "(x_n)" sequence

Paul Nevai

Abstract

I will discuss (generalizations of) "(x_n)" sequence, or, in other words, the recurrence coefficients for the orthogonal polynomials associated with the weight function $\exp(-c/4x^4 - K/2x^2)$ on the real line that have some fascinating properties that have long thrilled and puzzled me. In certain fancy circles the equation describing this sequence is called Discrete Painlevé Equation #1 (googleable) but in our down-to-earth universe it is just a non-linear second order difference equation with mystic but irresistible beauty and attraction.

Paul Nevai,
KAU, Jeddah, Saudi Arabia,
Upper Arlington (Columbus), Ohio, USA.
paul@nevai.us

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

Recent progress on quasi-interpolants derived from some classical linear approximation operators

Paul Sablonnière

Abstract

In this survey paper are presented some properties and applications of new quasi-interpolants derived from the representations of Bernstein, Baskakov, Szász-Mirakyan, Weierstrass and De la Vallée Poussin operators as linear differential operators on algebraic or trigonometric polynomials. In particular, we study their infinite norms, their convergence properties and their ability to generating useful quadrature or differentiation rules. This method also works for other families of linear approximation operators.

Paul Sablonnière,
INSA & IRMAR,
Centre de Mathématiques, INSA de Rennes,
20 avenue des Buttes de Coësmes, CS 70839,
F-35708 Rennes Cedex 7, France.
psablonn@insa-rennes.fr

Convergence of multivariate interpolation by increasingly flat translation kernels with finite smoothness

Yeon Ju Lee, Charles A. Micchelli and Jungho Yoon

Abstract

This study is concerned with the behavior of multivariate interpolation by finitely smooth kernel function as the kernel is increasingly flat. First, interpolation by radial basis function (RBF) is considered. We show that when the basis function is scaled to be increasingly flat, the corresponding interpolants converge to a polyharmonic spline interpolant for a larger class of RBFs including the Sobolev splines (of arbitrary order) as well as the Wendland compactly supported RBFs. Second, we improve upon some observations made in recent papers on the subject of increasingly flat interpolation. We shall establish that the corresponding Lagrange functions converges both for a finite set of functions (collocation matrix) and also for kernels (Fredholm matrix). In our analysis, we use a finite Maclaurin expansion of a multivariate function with remainder and some additional matrix theoretic facts.

Keywords: multivariate kernel interpolation, radial basis function, polyharmonic spline, collocation matrix.

AMS Classification: 41A05, 41A15, 41A25, 41A30, 41A63.

Bibliography

[1] **E. Larsson and B. Fornberg**, Theoretical and computational aspects of multivariate interpolation with increasingly flat radial basis functions, *Comp. Math. Appl.* 49 (2005) 103–130.

[2] **Y. Lee and C. A. Micchelli**, Collocation matrices for interpolation and approximation, J. Appr. Theory **174** (2013) 148–181.

[3] **Y. Lee, C. A. Micchelli and J. Yoon**, On convergence of flat multivariate interpolation by translation kernels with finite smoothness, *Constr. Approx* (2014) (to appear).

[4] **Y. Lee, G. Yoon and J. Yoon**, Convergence of increasingly flat radia basis interpolants to polynomial interpolants, *SIAM J. Math. Anal.* 39 (2007) 537–553.

[5] **R. Schaback**, Multivariate Interpolation by Polynomials and Radial Basis Functions, *Constr. Approx.* 21 (2005) 293–317.

[6] **G. Song, J. Riddle, G. Fasshauer and F. J. Hickernell**, Multivariate Interpolation with Increasingly Flat Radial Basis Functions of Finite Smoothness, *Advances in Comp. Math.* 293–317 (to appear).

[7] **H. Wendland**, *Scattered Data Approximation*, Cambridge Monographs on Applied and Computational Mathematics, Cambridge University Press, Cambridge, UK, 2005.

Yeon Ju Lee,
Institute of Mathematical Sciences,
Ewha W. University,
Seoul 120-750, Republic of Korea.
leeyeonju08@gmail.com

Charles A. Micchelli,
Department of Mathematics and Statistics,
State University of New York,
The University at Albany,
Albany, New York 12222, USA.
leeyeonju08@gmail.com

Jungho Yoon,
Department of Mathematics,
Ewha W. University,
Seoul 120-750, Republic of Korea.
yoon@ewha.ac.kr

Short Talks/Posters

Approximating functions of bounded variation by positive linear operators*

José A. Adell, J. Bustamante and J.M. Quesada

Abstract

Let $(L_n, n \geq 1)$ be a sequence of positive linear operators allowing for a probabilistic representation of the form

$$L_n f(x) = Ef\left(x + \frac{Z_n(x)}{\sqrt{n}}\right), \quad x \in I, \quad n \geq 1,$$

where I is a real interval, $f : I \to \mathbb{R}$ is a measurable function and $Z_n(x)$ is an appropriate random variable. Denote by $B_x(I)$ the set of bounded measurable functions having right and left limits at x.

The aim of this talk is to give explicit estimates both for

$$L_n f(x) - \frac{f(x+) + f(x-)}{2} \quad \text{and} \quad L_n \phi(x) - \phi(x),$$

for any $f \in B_x(I)$ and for any absolutely continuous function ϕ with Radon-Nikodym derivative $\phi' \in B_x(I)$, respectively. This setting includes the case when f or ϕ' have bounded variation on I, respectively. Such estimates are obtained under the following two assumptions: the tail probabilities of $Z_n(x)$ are exponentially bounded, and

$$\lim_{n\to\infty} P(Z_n(x) > 0) = \lim_{n\to\infty} P(Z_n(x) < 0) = \frac{1}{2}.$$

These (nonessential) assumptions have been chosen because the operators usually considered fulfil them. In our approach, a crucial tool to obtain explicit upper bounds is the notion of median of a random variable.

The main results are applied to the Bernstein polynomials, as well as to certain convolution operators of exponential type which show that our estimates cannot be essentially improved.

Keywords: bounded variation, positive linear operator, exponential tail, median, Bernstein polynomials.

AMS Classification: 41A36, 60E05.

*This work has been partially supported by research grants MTM2011-23998 and DGA E-64, by Junta de Andalucía Research Group FQM268, and by FEDER funds.

Bibliography

[1] **J. A. Adell, J. Bustamante, and J.M. Quesada**, Approximating functions of bounded variation by positive linear operators (preprint).

[2] **R. Bojanic and F. Cheng**, Rate of convergence of Bernstein polynomials for functions with derivatives of bounded variation, *J. Math. Anal. Appl.* **141** (1989) 136–151.

[3]**J. Bustamante, M.A. García Muñoz and J. M. Quesada**, Berstein Polynomials and discontinuous functions, *J. Math. Anal. Appl.* **411** (2014) 829–837.

[4] **H. Karsli, V. Gupta and A. Izgi**, Rate of pointwise convergence of a new kind of gamma operators for functions of bounded variation, *Appl. Math. Lett.* **22** (2009) 505–510.

[5] **X. M. Zeng and F. Cheng**, First order absolute moment of Meyer-König and Zeller operators and their approximation for some absolutely continuous functions, *Math. Slovaca* **61** (2011) 1–10.

[6] **X. M. Zeng and A. Piriou**, On the rate of convergence of two Bernstein-Bézier type operators for bounded variation functions, *J. Approx. Theory* **95** (1998) 369–387.

José A. Adell,
Departamento de Métodos Estadísticos,
Universidad de Zaragoza, Zaragoza 50009, Spain.
adell@unizar.es

Jorge Bustamante,
Facultad de Ciencias Físico-Matemáticas,
Benemérita Universidad Autónoma de Puebla, México.
jbusta@fcfm.buap.mx

J. M. Quesada,
Departamento de Matemáticas,
Universidad de Jaén.
jquesada@ujaen.es

Chebyshev blossoming and Gaussian quadrature rules*

Rachid Ait-Haddou

Abstract

In this talk we give a characterization of the nodes and weights of Gaussian quadrature rules in extended Chebyshev spaces [2] in terms of their associated Chebyshev blossoms[3]. Applications to Gaussian quadrature rules in Muntz spaces [1] will be discussed.

Keywords: Chebyshev blossoming, Gaussian quadrature rules, Muntz spaces, Schur functions.

AMS Classification: 41A28, 41A40, 41A60.

Bibliography

[1] **R. Ait-Haddou, Y. Sakane and T. Nomura**, Chebyshev blossoming in Muntz spaces: toward shaping with Young diagrams, *J. Comput. Appl. Math.* **247** (2013) 172–208.

[2] **S. J. Karlin and W. J. Studden**, *Tchebycheff Systems*, Interscience, New York, 1966.

[3] **M.-L. Mazure**, Blossoming: A geometrical approach, *Constr. Approx.* **15** (1999) 33–68.

Rachid Ait-Haddou,
Visual Computing Center,
King Abdullah University of Science and Technology,
Thuwal 23955-6900, Saudi-Arabia.
rachid.aithaddou@kaust.edu.sa

*This work has been supported by the KAUST Visual Computing Center.

Rates of convergence for a Kantorovich-Stancu generalization of Szasz type operators

Rabia Aktaş, Bayram Çekim and Fatma Taşdelen Yeşildal

Abstract

In the present paper, we study the rates of convergence for Kantorovich-Stancu type generalization of Szasz operators including some known polynomials. The special cases of these type operators are indicated and their approximation properties are also discussed.

Keywords: Szasz operator, modulus of continuity, rate of convergence, Brenke type polynomials, Gould-Hopper polynomials, voronovskaya type theorem.
AMS Classification: 41A25, 41A36.

Bibliography

[1] **F. Altomare and M. Campiti**, *Korovkin-type approximation theory and its applications.* Appendix A by Michael Pannenberg and Appendix B by Ferdinand Beckhoff, de Gruyter Studies in Mathematics, 17. Walter de Gruyter & Co., Berlin, 1994.

[2] **D. Barbosu**, Kantorovich-Stancu Type Operators, *J. Ineq. Pure and Appl. Math.* **5(3)** (2004), Art. 53.

[3] **P. L. Butzer**, On the extensions of Bernstein polynomials to the infinite interval, *Proc. Amer. Soc.* **5** (1954), 547-553.

[4] **T.S. Chihara**, *An Introduction to Orthogonal Polynomials*, Gordon and Breach, New York, 1978.

[5] **Z. Ditzian and V. Totik**, *Moduli of Smoothness*, Springer-Verlag, New York, 1987.

[6] **H. W. Gould and A. T. Hopper**, Operational formulas connected with two generalizations of Hermite polynomials, *Duke Math. J.* **29** (1962), 51-63.

[7] **M.E.H. Ismail**, On a generalization of Szász operators, *Mathematica (Cluj)* **39** (1974), 259-267.

[8] **A. Jakimovski and D. Leviatan**, Generalized Szasz operators for the approximation in the infinite interval, *Mathematica (Cluj)* **11** (1969), 97-103.

[9] **O. Szasz**, Generalization of S. Bernstein's polynomials to n the infinite interval, *J. Research Nat. Bur. Standards* **45** (1950), 239-245.

[10] **F. Taşdelen, R. Aktaş and A. Altın**, A Kantorovich Type of Szasz Operators Including Brenke-Type Polynomials, *Abstract and Applied Analysis* **2012** (2012), 13 pages.

[11] **S. Varma, S. Sucu and G. İçöz**, Generalization of Szasz operators involving Brenke type polynomials, *Comput. Math. Appl.* **64 (2)** (2012), 121-127.

[12] **S. Varma, G. İçöz and S. Sucu**, On Some Extensions of Szasz Operators Including Boas-Buck-Type Polynomials, *Abstract and Applied Analysis* **2012** (2012), 15 pages.

Rabia Aktaş,
Ankara University, Faculty of Science,
Department of Mathematics,
06100 Tandoğan, Ankara/ Turkey.
raktas@science.ankara.edu.tr

Bayram Çekİm,
Gazi University,
Faculty of Science,
Department of Mathematics,
06500 Beşevler, Ankara/ Turkey.
bayramcekim@gazi.edu.tr

Fatma Taşdelen Yeşİldal,
Ankara University, Faculty of Science,
Department of Mathematics,
06100 Tandoğan, Ankara/ Turkey.
tasdelen@science.ankara.edu.tr

On quasi–orthogonal polynomials in several variables

M. Alfaro, A. Peña, T.E. Pérez and M.L. Rezola

Abstract

Let $\{\mathbb{P}_n\}_{n\geq 0}$ and $\{\mathbb{Q}_n\}_{n\geq 0}$ be two monic polynomial systems in several variables. Whenever both polynomial systems are orthogonal, the existence of a linear structure relation

$$\mathbb{Q}_n = \mathbb{P}_n + M_n \mathbb{P}_{n-1}, \quad n \geq 1,$$

with M_n constant matrices, is characterized in terms of the orthogonality moment functionals. Moreover, assuming that one of the polynomial systems is orthogonal, we study when the other one is also orthogonal. Some illustrative examples are presented.

Keywords: multivariate orthogonal polynomials, three term relations, moment functionals.
AMS Classification: 42C05, 33C50.

Manuel Alfaro,
Departamento de Matemáticas and IUMA,
Universidad de Zaragoza,
Zaragoza, Spain.
alfaro@unizar.es

Ana Peña,
Departamento de Matemáticas and IUMA,
Universidad de Zaragoza,
Zaragoza, Spain.
anap@unizar.es

Teresa E. Pérez,
Departamento de Matemática Aplicada,
Facultad de Ciencias,
Universidad de Granada,
18071 Granada, Spain.
tperez@ugr.es

María Luisa Rezola,
Departamento de Matemáticas and IUMA,
Universidad de Zaragoza,
Zaragoza, Spain.
rezola@unizar.es

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

On approximation schemes and compactness

A. G. Aksoy and J. M. Almira

Abstract

One of the basic notions in functional analysis is compactness. Its utility has become of fundamental importance after the appearance of Arzelà-Ascoli's Theorem [13], [14] especially pointing its use for the proof of existence results when investigating the solutions of differential equations. Indeed, a key step for the proof of convergence in many algorithms is precisely to show that a certain set is compact, and many theorems have been produced to characterize compactness of subsets of the numerous function spaces and operator spaces that appear in functional analysis. The compactness of operators was also a main ingredient for the study of the solutions of integral equations, and was indeed introduced by Hilbert in his studies of the equations of Mathematical Physics. In particular, Hilbert and his student Schmidth proved a very nice decomposition formula for all self-adjoint compact operator $T: H \to H$, where H is any separable Hilbert space: the spectral decomposition theorem. This theory was soon investigated and amplified to a beautiful set of results which we call nowadays Riesz theory (or Riesz-Schauder Theory) and is devoted to the study of operators $S : X \to X$ (where X denotes any complex Banach space) that can be expressed as $S = \lambda I_X - T$ with $\lambda \neq 0$ (an scalar) and $T : X \to X$, a compact operator. In such study, the spectral properties of the operator T are essential and, in connection with these properties, it was soon discovered that some entropy and approximation quantities were of great importance (see, e.g., [21] for a detailed study of this connection). Compactness has also been a fundamental concept for the development of other parts of Mathematical Analysis, such as Fixed Point Theory or Approximation Theory. Concretely, Brouwer's fixed point theorem [18] asserts that every compact convex set K in $\mathbb{R}^n$ is a fixed point space, that is, if $f : K \to K$ is continuous, then $f(x) = x$ for some $x \in K$ (see [38, p. 25] for a nice easy demonstration). On the other hand, Schauder's fixed point theorem [51], which has numerous applications in Mathematical Analysis, asserts that every convex set in a normed linear space is a fixed point space for compact maps (see also [16]). Among the results equivalent to Brouwer's fixed point theorem, the theorem of Knaster, Kuratowski and Mazurkiewicz (in short, KKM) [36] occupies a special place. Ky Fan, using KKM maps, was able to prove a best approximation theorem [29]. Later on, this concept was generalized by Khamsi to metric space setting by demonstrating a result which can be seen as an extension of Brouwer and Schauder's fixed point theorems (see [35]). Finally, just to include in this section some

results related to Approximation Theory, we would like to stand up that compactness of natural embeddings $Y \hookrightarrow X$ is, in fact, the main reason because, in many classical contexts, we can prove that approximation errors (with respect to arbitrary approximation schemes) and Fourier coefficients of functions that belong to the space Y, decay to zero with a certain prescribed behavior. This was recently proved by Almira and Oikhberg [12] and by Almira [8].

In this address, we survey some results about the characterization of compactness in which the concept of approximation scheme has had a role. Concretely, we present several results that were proved by the second author, jointly with Luther, a decade ago, when these authors were working on a very general theory of approximation spaces [9], [10] (see also [31]) and we also introduce and show the basic properties of a new concept of compactness, which was studied by the first author in the eighties [1], [2], [3], [6], by using a generalized concept of approximation scheme and its associated Kolmogorov numbers, which generalizes the classical concept of compactness..

Keywords: Approximation schemes, Compactness.
AMS Classification: 41A65, 41A25.

Bibliography

[1] **A. Aksoy,** *Approximation Schemes, Related s-Numbers and Applications,* Ph. D. Thesis, University of Michigan, 1984.

[2] **A. Aksoy,** Intermediate Spaces, *Turkish Journal of Mathematics* **13** (3) (1989) 79-90.

[3] **A. Aksoy,** Q-compact Sets and Q-compact Maps, *Mathematica Japonica* **36** (1) (1991), 1-7.

[4] **A. Aksoy,** A Generalization of n-widths, in *Approximation Theory, Spline Functions and Applications,* Kluwer Academic Publishers, NATO-ASI Series, 1992, 269-278.

[5] **A. Aksoy and G. Lewicki,** Diagonal Operators, s-Numbers and Bernstein Pairs, *Note Mat.* **17** (1999) 209-216.

[6] **A. Aksoy and M. Nakamura,** The Approximation Numbers $a_n(T)$ and $\mathbf{Q}$-precompactness, *Mathematica Japonica* **31** (6) (1986) 827-840.

[7] **J. M. Almira,** On strict inclusion relations between approximation and interpolation spaces, *Banach J. Math. Anal.* **5** (2) (2011) 93-105

[8] **J. M. Almira,** A simple observation about compactness and fast decay of Fourier coefficients, *Annals of Functional Analysis* **1** (1) (2010) 41-43.

[9] **J. M. Almira and U. Luther,** Generalized approximation spaces and applications, *Math. Nachr.* **263-264** (2004) 3-35.

[10] **J. M. Almira and U. Luther,** Compactness and generalized approximation spaces, *Numer. Funct. Anal. Optim.* **23** (2002) 1–38.

[11] **J. M. Almira and T. Oikhberg,** Approximation schemes satisfying Shapiro's theorem, *J. Approx. Theory* **164** (2012) 534-571.

[12] **J. M. Almira and T. Oikhberg,** Shapiro's theorem for subspaces, *J. Math. Anal. Appl.* **388** (2012) 282-302.

[13] **C. Arzelà,** Un'osservazione intorno alle serie di funzioni, *Rend. Dell' Accad. R. Delle Sci. Dell'Istituto di Bologna* (1882-1883) 142-159.

[14] **G. Ascoli,** Le curve limiti di una varietà data di curve, Memoria "Acc. dei Lincei ", Memorie della Cl. Sci. Fis. Mat. Nat. 18 (3) (1884) 521-586.

[15] **S. N. Bernstein,** Sur le probleme inverse de la théorie de la meilleure approximation des functions continues. *Comtes Rendus* **206** (1938) 1520-1523 (see also: Ob obratnoi zadache teorii nailuchshego priblizheniya nepreryvnykh funksii, Sochineniya Vol II (1938)).

[16] **F. F. Bonsall,** *Lectures on some fixed point theorems of functional analysis,* Tata Institute Of Fundamental Research, Bombay, 1962.

[17] **P. A. Borodin,** On the existence of an element with given deviations from an expanding system of subspaces, *Mathematical Notes* 80 (5) (2006) 621-630 (Translated from Matematicheskie Zameti 80 (5) (2006) 657-667).

[18] **L. E. J. Brouwer,** Über Abbildungen von Mannigfaltigkeiten, *Math. Ann.* **71** (1911) 97-115.

[19] **Y. Brudnyi and N. Kruglyak,** On a family of approximation spaces, In: *Investigations in function theory of several real variables* Yaroslavl' State Univ., Yaroslavl' (1978),15-42.

[20] **P. L. Butzer and K. Scherer,** *Approximationsprozesse und Interpolationsmethoden*, Bibliographisches Institut Mannheim, Mannheim, 1968.

[21] **B. Carl and I Stephani,** *Entropy, compactness and the approximation of operators*, Cambridge University Press, 1990.

[22] **F. Cobos,** On the Lorentz-Marcinkiewicz operator ideals, *Math. Nachr.* **126** (1986) 281-300.

[23] **F. Cobos and M. Milman,** On a limit class of approximation spaces, *Numer. Funct. Anal. Optim.* **11** 1-2 (1990) 11-31

[24] **F. Cobos and I. Resina,** Representation theorems for some operator ideals, *J. London Math. Soc.* **39** (1989) 324-334.

[25] **R. A. DeVore,** Nonlinear approximation, *Acta Numer.* **7** (1998) 51-150.

[26] **R. A. DeVore and G. G. Lorentz,** *Constructive approximation,* Springer, 1993.

[27] **E. Dubinsky and M.S. Ramanujan,** *On λ-nuckearity,* Mem. Amer. Math. Soc.,**128**, 1972.

[28] **N. Dunford and J. T. Schwartz**, *Linear Operators, Part I* Interscience publishing, New York/ London, 1958.

[29] **K. Fan,** A generalization of Tychonoff's fixed point theorem, *Math. Ann.* **142** (1960/1961) 305-310.

[30] **F. Feher and G. Grässler,** On an extremal scale of approximation spaces, *J. Comp. Anal. Appl.* **3** (2) (2001), 95-108.

[31] **M. A. Fugarolas,** Compactness in Approximation Spaces, *Colloquium Mathematicum* **64** (2) (1994) 253-262.

[32] **H. Hanche-Olsen and H. Holdena,** The Kolmogorov-Riesz compactness theorem, *Expositiones Mathematicae* **28** (4) (2010) 385-394.

[33] **C. V. Hutton, J. S. Morrell and J. R. Retherford,** Approximation Numbers and Kolmogoroff Diameters of Bounded Linear Operators, *Bull. Amer. Math. Soc.* **80** (1974) 462-466.

[34] **C. V. Hutton, J. S. Morrell and J. R. Retherford,** Diagonal Operators, Approximation Numbers and Kolmogoroff Diameters, *J. Approx. Theory* **16** (1976) 48-80.

[35] **M. A. Khamsi,** KKM and Ky Fan theorems in hyperconvex metric spaces, *J. Math Anal. Appl.* **204** (1996) 298-306.

[36] **B. Knaster, C. Kuratowski and S. Mazurkiewicz,** Ein Beweis des Fixpunktsatzes für n-dimensionale Simplexe, *Fund. Math.* **14** (1929) 132-137.

[37] **A. N. Kolmogorov,** Über Kompaktheit der Funktionenmengen bei der Konvergenz im Mittel, *Nachr. Ges. Wiss. Göttingen* **9** (1931), 6063. English translation: On the compactness of sets of functions in the case of convergence in the mean, in V. M. Tikhomirov (ed.), *Selected Works of A. N. Kolmogorov, Vol. I,* Kluwer, Dordrecht, 1991, 147-150.

[38] **J. Matousek,** *Using the Borsuk-Ulam Theorem,* UniversityText, Springer, 2003.

[39] **T. Oikhberg,** Rate of Decay of s-Numbers, *J. Approx. Theory* **163** (2011) 311-327.

[40] **J. Peetre and G. Sparr,** Interpolation of normed abelian groups, *Annali di Matematica Pura ed Applicata* **12** (1972)

[41] **A. Pietsch,** Approximation spaces, *Journal of Approximation Theory* **32** (1981) 115–134.

[42] **A. Pietsch,** *Operator ideals*, North-Holland, Amsterdam, 1980.

[43] **A. Pietsch,** Bad Properties of the Bernstein Numbers, *Studia Math.* **184** (2008) 263-269.

[44] **A. Plichko,** Rate of Decay of the Bernstein Numbers, *Journal of Mathematical Physics*, Analysis, Geometry. **9** (1) (2013) 59-72.

[45] **G. Pisier,** Counterexamples to a Conjecture of Grothendieck, *Acta Math.* **151** (1983) 181-208.

[46] **E. Pustylnik,** Ultrasymmetric sequence spaces in approximation theory, *Collectanea Mathematica* **57** (3) (2006) 257-277.

[47] **E. Pustylnik,** A new class of approximation spaces, *Rend. Circ. Mat. Palermo* Ser. II, Suppl., v. 76 (2005), 517-532.

[48] **S. Rolewicz,** *Metric linear spaces, 2th Ed.* Mathematics and its applications, East European Series, Kluwer Acad. Publ., 1985.

[49] **W. Rudin,** *Functonal Analysis (2 Edition),* Mc-Graw Hill, Inc., 1991.

[50] **H. S. Shapiro,** Some negative theorems of Approximation Theory, *Michigan Math. J.* **11** (1964) 211–217.

[51] **J. Schauder,** Der Fixpunktsatz in Funktionalräumen, *Studia Math.* **2** (1930), 171-180.

[52] **J. Simon,** Compact sets in the space $L^p(0,T,B)$, *Ann. Math. Pura Appl.* **146** (4) (1987) 65-96.

[53] **I. Singer,** *Best approximation in normed linear spaces by elements of linear subspaces*, Springer Verlag, New York, 1970.

[54] **T. Terzioğlu,** A characterization of compact linear mappings, Arch. Math. (Basel) 22 (1971) 76-78.

[55] **N. Ţiţa,** Equivalent quasi-norms on some operator ideals, Annal. Univ. Craiova **28** (2001) 16-23.

[56] **N. Ţiţa,** On a limit class of Lorentz-Zygmund ideals, Analysis, Functional Equations, Approximation and Convexity, 302–306, Cluj Napoca, 1999.

[57] **N. Ţiţa,** A generalization of the limit class of approximation spaces, *Annal. Univ. Iasi* **43** (1997) 133-138.

[58] **N. Tiţa,** Approximation spaces and bilinear operators, *Studia Univ. Babes-Bolyai* Ser. Math. **35** (4) (1990) 89-92.

[59] **N. Tiţa,** On a class of $\ell_{\Phi,\phi}$ operators, *Collectanea Mathematica* 32 (3) (1981) 275-279.

[60] **N. Tiţa,** $L_{\Phi,\phi}$ operators and (Φ, ϕ) spaces, *Collectanea Mathematica*, 30 (1) (1979) 3-10.

[61] **I. S. Tjuriemskih,** On a problem of S. N. Bernstein, *Uchen. Zap. Kalinin. Gos. Ped. Inst.* **52** (1967) 123–129 (Russian).

[62] **I. S. Tjuriemskih,** B property of Hilbert spaces, *Uchen. Zap. Kalinin. Gos. Ped. Inst.* **39** (1964) 53–64 (Russian).

A. G. Aksoy,
Department of Mathematics. Claremont McKenna College,
Claremont, CA, 91711, USA.
aaksoy@cmc.edu

J. M. Almira,
Departamento de Matemáticas. Universidad de Jaén,
E.P.S. Linares, C/Alfonso X el Sabio, 28,
23700 Linares (Jaén) Spain.
jmalmira@ujaen.es

On the rate of convergence of modified Baskakov type operators

Sevilay Kırcı Serenbay and Didem Aydın Arı

Abstract

In this paper, we estimate the rate of convergence of modified Baskakov type operators with derivatives of bounded variation.

Keywords: Baskakov type operators, bounded variation, total variation, rate of convergence. **AMS Classification:** 41A35, 41A36.

Bibliography

[1] **Ç. Atakut, S.K Serenbay and İ. Büyükyazıcı**, Approximation properties for generalized Baskakov-type operators, *Mathematical Sciences* **7:21** (2013).

[2] **V.A Baskakov**, An example of a sequence of linear positive operators in the space of continuous functions, *Dokl. Akad. Naulc* **113** (1957) 249–251.

[3] **Z.R. Guo and D.X. Zhou**,Approximation theorems for modified Szász operators, *Acta Sci. Math.* **56** (1193) 3-4, 311–321.

Sevilay KırcıSerenbay,
Bakent University, Faculty of Education,
Department of Mathematics Education,
Ankara, Turkey.
sevilaykirci@gmail.com

Didem Aydın Arı,
Kırıkkale University, Faculty of art and science,
Department of Mathematics,
Ankara, Turkey.
didem@kku.edu.tr

On differential quadrature*

D. Barrera, P. González, F. Ibáñez and M. J. Ibáñez

Abstract

The Differential Quadrature Method (DQM) is a numerical discretization technique for the approximation of derivatives by means of weighted sums of function values. It was introduced by Bellman and coworkers in the early 1970's, and it has been extensively employed to approximate spatial partial derivatives (cf. [1], [7] and references quoted therein). The classical DQM is polynomial-based, but some spline based DQMs have been proposed to overcome the limitation concerning the number of grid points involved. Given a B-spline (cf. [2], [8]), a cardinal lagrangian o hermitian spline with a compactly supported fundamental function is defined, from which the approximation of the derivatives is obtained. The construction of these spline interpolants depends strongly on the degree of the B-spline (see for instance [3] and [9]). In this work we present a DQM based on interpolation and quasi-interpolation. Firstly, we consider the construction of compactly supported cardinal functions L based on B-splines such that

$$L(j) = \delta_{j,0}, \ j \in \mathbb{Z},$$

δ being the Kronecker sequence (cf. [6]). Then, we revise some spline discrete quasi-interpolants defined from the same B-splines (cf. e.g. [2], [8] and references quoted therein). Finally, both the interpolants and the quasi-interpolants are used to define boolean sum based interpolants (cf. [4]) having compactly supported fundamental functions again, and the maximal order of approximation. The quintic case is described (cf. [5]) and compared with the results obtained in [9].

Keywords: differential quadrature, B-spline, interpolation, discrete quasi-interpolation, boolean sum.

AMS Classification: 41A05, 41A15, 65D05, 65D07.

Bibliography

[1] **C. W. Bert and M. Malik**, Differential quadrature method in computational mechanics: a review, *Appl. Rev.* **49** (1996) 1–27.

*University of Granada and Junta de Andalucía

[2]**C. de Boor**, *A practical guide to splines*, Springer-Verlag, New York, 2001.

[3]**Q. Guo and H. Zhong**, Non-linear vibration analysis by a spline-based differential quadrature method, *Journal of Sound and Vibration* **269** (2004) 413–420.

[4]**J. Hoshek and D. Lasser**, *Fundamentals of Computer Aided Geometric Design*, A. K. Peters, Wellesley, Massachussets, 1993.

[5]**M. J. Ibáñez-Pérez**, Chebyshev-type discrete quasi-interpolants, *Mathematics and Computers in Simulation* 77 (2008) 218–227.

[6]**C. A. Micchelli**, Banded matrices with banded inverses, *J. Comput. Appl. Math.* **41** (1992) 281–300.

[7]**C. Shu**, *Differential Quadrature and its applications in Engineering*, Springer-Verlag, London, 2000.

[8]**L. L. Schumaker** *Spline functions. Basic theory*, John Wiley & Sons, New York, 1981.

[9]**H. Zhong**, Spline-based differential quadrature for fourth order differential equations and its applications to Kirchhoff plates, *Applied Mathematical Modelling* **28** (2006) 353–366.

D. Barrera,
Department of Applied Mathematics,
University of Granada, 18071-Granada, Spain.
`dbarrera@ugr.es`

P. González,
Department of Applied Mathematics,
University of Granada, 18071-Granada, Spain.
`prodelas@ugr.es`

F. Ibáñez,
Grupo Sacyr-Vallehermoso,
Paseo de la Castellana, 83, 28046-Madrid, Spain.
`fibanez@gruposyv.com`

M. J. Ibáñez,
Department of Applied Mathematics,
University of Granada, 18071-Granada, Spain.
`mibanez@ugr.es`

Durrmeyer type operators with respect to arbitrary measure*

Elena E. Berdysheva

Abstract

We consider Bernstein-Durrmeyer operators on a d-dimensional simplex as well as Szász-Mirakjan-Durrmeyer and Baskakov-Durrmeyer operators on the half-line $[0,\infty)$ with respect to an arbitrary measure. A motivation for this generalization comes from learning theory. The construction, given below for simplicity in the one-dimensional case, is the following.

Let $c = -1$, 0, or 1. The cases $c=-1$, $c=0$, and $c=1$ correspond to the one-dimensional Bernstein-Durrmeyer operator, Szász-Mirakjan-Durrmeyer operator, and Baskakov-Durrmeyer operator, respectively. Let $I_{-1}=[0,1]$, and $I_0=I_1=[0,\infty)$. The basis functions are defined on I_c by the formulae

$$p_{n,k}^{[c]}(x) = \begin{cases} \binom{n}{k} x^k (1-x)^{n-k}, & c=-1, \\ \frac{(nx)^k}{k!} e^{-nx}, & c=0, \\ (-1)^k \binom{-n}{k} x^k (1+x)^{-n-k}, & c=1. \end{cases}$$

Let ρ be a non-negative locally bounded Borel measure on I_c. The Durrmeyer type operators with respect to measure ρ are defined for functions f on I_c by

$$\mathbf{M}_{n,\rho}^{[c]} f := \sum_{k=0}^{\infty} \frac{\int_{I_c} f(t)\, p_{n,k}^{[c]}(t)\, d\rho(t)}{\int_{I_c} p_{n,k}^{[c]}(t)\, d\rho(t)}\, p_{n,k}^{[c]}$$

under some natural conditions on ρ and f.

In the talk, we concentrate on the convergence of the operators. We prove pointwise convergence at each point $x \in \operatorname{supp}\rho$ of continuity of f. Moreover, the convergence is uniform in every compact set $A \subset (\operatorname{supp})^\circ$ when f is continuous in A. Further on, we prove convergence in the weighted L^p-spaces, $1 \le p < \infty$, and give estimates for the rate of L^p-convergence.

Parts of the talk are based on joint work with Kurt Jetter, Bing-Zheng Li, and Eman Al Aidarous.

*The research was partially supported by KAU grant No. 20-130/1433 HiCi.

Keywords: Durrmeyer type operators, uniform convergence, L^p-convergence.
AMS Classification: 41A36, 41A63.

Elena Berdysheva,
German University of Technology in Oman (GUtech),
Muscat, Sultanate of Oman.
`elena.berdysheva@gutech.edu.om`

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

A hypercomplex approach to orthogonal polynomials in arbitrary dimension*

I. Cação, M.I. Falcão and H. Malonek

Abstract

The aim of the talk is to provide an alternative approach to the construction of orthogonal polynomials in arbitrary dimension by using hypercomplex function theory techniques. Hypercomplex function theory (or Clifford analysis) can be viewed as a generalization to higher dimensions of the theory of holomorphic functions of one complex variable by using Clifford algebras. In this framework the analogue of holomorphic functions is obtained as null-solutions to a generalized Cauchy-Riemann system and are usually called monogenic.

We construct orthogonal monogenic polynomials that are multiples of their (hypercomplex) derivatives, i.e., that form Appell sequences. The different representations obtained for these polynomials lead to some new features, namely multiplicative and inductive processes to obtain orthogonal bases of the space of monogenic polynomials of some fixed (homogeneous) degree.

Keywords: Clifford analysis, orthogonal bases, generalized Appell polynomials.
AMS Classification: 30G35, 35C11, 32A05.

I. Cação,
Center for Research and Development in Mathematics and Applications,
Department of Mathematics,
University of Aveiro,
Campus Universitário de Santiago,
3810-193 Aveiro, Portugal.
isabel.cacao@ua.pt

M. I. Falcão,
Department of Mathematics and Applications,
University of Minho,

*This work was supported by Portuguese funds through the CIDMA - Center for Research and Development in Mathematics and Applications, the Research Centre of Mathematics of the University of Minho and the Portuguese Foundation for Science and Technology ("FCT - Fundação para a Ciência e a Tecnologia"), within projects PEst-OE/MAT/UI4106/2014 and PEstOE/MAT/UI0013/2014.

Campus de Gualtar, 4710-057 Braga,
Portugal.
mif@math.uminho.pt

H. R. Malonek,
Center for Research and Development in Mathematics and Applications,
Department of Mathematics,
University of Aveiro,
Campus Universitário de Santiago, 3810-193 Aveiro,
Portugal.
hrmalon@ua.pt

Relation between Jackson's and Hahn's quantum operators*

José Luis Cardoso

Abstract

For $0 < q < 1$, $\omega \geq 0$, $\omega_0 := \omega/(1-q)$, and I a set of real numbers, the Hahn operator acting on a function $f : I \to \mathbb{R}(\mathbb{C})$ is defined by

$$D_{q,\omega}[f](x) := \frac{f(qx+\omega) - f(x)}{(q-1)x+\omega}\,, \quad x \in I \setminus \{\omega_0\}\,.$$

Its inverse operator is given in terms of the so-called Jackson-Thomae $(q,\omega)-$integral, also called Jackson-Nörlund $(q,\omega)-$integral. For $\omega = 0$ one obtains the Jackson's $q-$operator, D_q, whose inverse operator is given in terms of the so-called Jackson $q-$integral. By establishing links between $D_{q,\omega}$ and D_q, as well as between the q and the (q,ω) integrals, we show how to obtain the properties of $D_{q,\omega}$ and the $(q,\omega)-$integral from the corresponding ones fulfilled by D_q and the $q-$integral. We also consider $(q,\omega)-$analogues of the Lebesgue spaces.

These results were motivated by our previous research works on basic Fourier series.

Acknowledgement: The results presented in this talk were obtained in collaboration with José Carlos Petronilho.

Keywords: Jackson $q-$integral, $q-$analogues, Jackson-Nörlund $(q,\omega)-$integral, $(q,\omega)-$Lebesgue spaces.

AMS Classification: 33E20, 33E30, 40A05, 40A10.

Bibliography

[1] **K. A. Aldwoah, A. B. Malinowska and D. F. M. Torres,** The power quantum calculus and variational problems, *Dyn. Contin. Discrete Impuls. Syst. Ser. B: Appl. Algorithms* **Vol 19**, (2012) no 1-2 93–116.

*CM-UTAD (FCT-Portugal).

[2] **M. H. Annaby, A. E. Hamza and K. A. Aldwoah,** Hahn difference operator and difference Jackson-Nörlund integrals, *Journal of Optimization Theory and Applications* **154** (2012) 133–153.

[3] **J. Bustoz and J. L. Cardoso**, Basic Analog of Fourier Series on a $q-$Linear Grid, *J. Approx. Theory* **112** (2001) 134–157.

[4] **J. L. Cardoso**, Basic Fourier series on a $q-$linear grid: convergence theorems, *J. Math. Anal. Appl.* **323** (2006) 313–330.

[5] **J. L. Cardoso**, Basic Fourier series: convergence on and outside the $q-$linear grid, *J. Fourier Anal. Appl.* **17**(1) (2011) 96–114.

[6] **J. L. Cardoso and J. Petronilho**, Variations around Jackson's quantum operator (submitted).

[7] **R. Floreanini R. and L. Vinet**, Quantum symmetries of q-difference equations, *J. Math. Phys* **36** (1995) 3134–3156.

[8] **A. B. Malinowska and D. F. M. Torres**, The Hahn quantum variational calculus, *J. Optim. Theory Appl.* **147** no. 3 (2010), 419–442.

José Luis Cardoso,
UTAD, Quinta de Prados,
Edifício das Ciências Florestais,
Departamento de Matemática, CM-UTAD,
5001-911 Vila Real, Portugal.
jluis@utad.pt

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

Rational positive quadrature formulas on the interval and on the unit circle*

Ruymán Cruz Barroso

Abstract

Let $\mathring{\mu}$ be a positive measure on the unit circle $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$. In this talk we consider positive (with positive weights) rational interpolatory quadrature formulas on the unit circle (see [1]-[3]) that approximate integrals of the form $I_{\mathring{\mu}}(f) = \int_{\mathbb{T}} f(z)d\mu(z)$. These rules, that are exact in spaces of rational functions, may have some of the nodes fixed in advance. The existence and some of its computational aspects will be discussed along with a connection with positive rational interpolatory quadrature formulas with prescribed nodes on the interval, that approximate integrals of the form $I_{\mu}(g) = \int_{-1}^{1} g(x)d\mu(x)$, where the measures μ and $\mathring{\mu}$ are related by the Joukowsky transformation. In addition, recent results on the computation of para-orthogonal rational functions via a three-term recurrence relation will be studied (see [4]).

Keywords: Rational Gaussian quadrature formulas, Rational Szegő-type quadrature formulas, orthogonal rational functions, para-orthogonal rational functions.
AMS Classification: 42C05, 65D32.

Bibliography

[1] **A. Bultheel, R. Cruz-Barroso, K. Deckers and F. Perdomo-Pío**, Positive rational interpolatory quadrature formulas on the unit circle and the interval, *Appl. Numer. Math.* **60** (2010), 1286–1299.

[2] **A. Bultheel, P. González-Vera, E. Hendriksen and O. Njåstad**, Computation of rational Szegő-Lobatto quadrature formulas, *Appl. Numer. Math.* **60** (2010) 1251–1263.

*Department of Mathematical Analysis, La Laguna University, Canary Islands, Spain. Supported by Dirección General de Programas y Transferencia de Conocimiento, Ministerio de Ciencia e Innovación of Spain under grant MTM2011-28781.

[3] **A. Bultheel, P. González-Vera, E. Hendriksen and O. Njåstad**, Rational quadrature formulas on the unit circle with prescribed nodes and maximal domain of validity, *IMA J. Numer. Anal.* **30** (2010) 940–963.

[4] **K. Castillo, R. Cruz-Barroso and F. Perdomo-Pío**, Para-orthogonality from a new viewpoint (submitted, 2014).

Ruymán Cruz Barroso,
Facultad de Matemáticas,
Calle Astrofísico Francisco Sánchez,
38271-La Laguna, Spain.
`rcruzb@ull.es`

Rate of convergence of generalized Favard-Szàsz type operators for functions of bounded variation

Özge Dalmanoğlu, İbrahim Büyükyazici and Ertan İbikli

Abstract

In this paper, we consider the modified Favard-Szàsz operators introduced by N. İspir and Ç. Atakut [4]. We give an estimate for the rate of convergence for functions of bounded variation on $[0, \infty)$.

Keywords: rate of convergence, bounded variation, Favard-Szàsz type operators.
AMS Classification: 41A25, 41A36.

Bibliography

[1] **V. Gupta and N. İspir**, On the Bezier variant of generalized Kantorovich type Balazs operators, *Appl. Math. Letters* **18 (5)** (2005) 1053-1061.

[2] **V. Gupta and R.P. Pant**, Rate of convergence for the modified Szàsz-Mirakyan operators on functions of bounded variation, *J. Math. Anal. Appl.* **233** (1999) 476–483.

[3] **G. G. Lorentz**, *Bernstein Polynomials*, Univ. of Toronto Press, Toronto, 1953.

[4] **N. İspir and Ç. Atakut**, Approximation by modified Szàsz–Mirakjan operators on weighted spaces, *Proc. Indian Acad. Sci.* **112(4)** (2002) 571–578.

[5] **X. M. Zeng**, On the rate of convergence of the generalized Szàsz type operators for functions of bounded variation, *J. Math. Anal. Appl.* **226** (1998) 309–325.

Özge Dalmanoğlu ,
Department of Mathematics Education,
Başkent University, Ankara, Turkey.
odalmanoglu@baskent.edu.tr

İbrahim Büyükyazici,
Department of Mathematics, Ankara University,
Ankara, Turkey.
`ibuyukyazici@gmail.com`

Ertan İbikli,
Department of Mathematics, Ankara University,
Ankara, Turkey.
`ibikli@.ankara.edu.tr`

Christoffel functions on Jordan curves with respect to measures with jump singularity

Tivadar Danka

Abstract

Let μ be a finite Borel measure on the complex plane with compact support. The n-th Christoffel function is defined by

$$\lambda_n(\mu, z) = \inf_{\deg(P_n)\leq n} \int \frac{|P_n(w)|^2}{|P_n(z)|^2} d\mu(w),$$

where the infimum is taken for all polynomials P_n such that $\deg(P_n) \leq n$. In this talk we study the asymptotic behaviour of Christoffel functions for a class of measures. Assume that μ belongs to the **Reg** class and its support is some Jordan curve γ. Let $z_0 \in \gamma$ and assume μ is absolutely continuous in a neighborhood of z_0 with respect to the arc length measure s_J. We show that if $d\mu(z) = w(z)ds_J(z)$, where w has a jump singularity at z_0 with left and right limits A and B, then we have

$$\lim_{n\to\infty} n\lambda_n(z_0, \mu) = \frac{ds_J(z_0)}{d\omega_\gamma} \frac{A - B}{\log A - \log B},$$

where ω_γ denotes the equillibrium measure with respect to γ.

Keywords: Christoffel function, asymptotic behavior, Jordan curve, equillibrium measure, Green function.
AMS Classification: 42C05.

Bibliography

[1] **A. Foulquié Moreno, A. Martinez-Finkelshtein and V. L. Sousa**, Asymptotics of orthogonal polynomials for a weight with a jump on [-1,1], *Constructive Approximation* **33** (2011) 219–263.

[2] **V. Totik**, Christoffel functions on curves and domains, *Transactions of the American Mathematical Society* **362** (2010) 2053–2087.

[3] **V. Totik**, The polynomial inverse image method, *Approximation Theory XIII: San Antonio 2010, Springer Proceedings in Mathematics, M. Neamtu and L. Schumaker* 345–367.

Tivadar Danka,
Bolyai Institute, University of Szeged,
Szeged, Aradi v. tere 1, 6720, Hungary.
`tdanka@math.u-szeged.hu`

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

Sobolev–type orthogonal polynomials on the unit ball

Antonia M. Delgado, Teresa E. Pérez and Miguel A. Piñar

Abstract

In this presentation, we consider the Sobolev–type inner product of the form

$$(p, q)_S = (p, q)_\mu + \lambda \sum_{k=0}^{N} \frac{\partial p(s_k)}{\partial n} \frac{\partial q(s_k)}{\partial n}, \quad \lambda > 0,$$

$(\cdot, \cdot)_\mu$ is the usual inner product on the unit ball B^d, and $\frac{\partial}{\partial n}$ represents the normal derivative on the sphere S^{d-1}. Then, multivariate orthogonal polynomials of Sobolev–type and the kernel functions associated with this Sobolev–type inner product are studied. More explicitly, we express them in terms of those corresponding with the original inner product. These results are applied to obtain the asymptotics of the Christoffel functions. Finally, the special case of the Sobolevtype modification of the bivariate classical measure on the unit disk obtained by adding the outward normal derivatives on a finite set of uniformly distributed points on the unit circle is presented.

Keywords: multivariate orthogonal polynomials, Sobolev inner products, Christoffel functions, asymptotics.

AMS Classification: 42C05, 33C50.

Antonia M. Delgado,
Dpto. de Matemática Aplicada, Universidad de Granada.
`amdelgado@ugr.es`

Teresa E. Pérez,
Dpto. de Matemática Aplicada, Universidad de Granada.
`tperez@ugr.es`

Miguel A. Piñar,
Dpto. de Matemática Aplicada, Universidad de Granada.
`mpinar@ugr.es`

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

Explicit expressions for generalized Hermite polynomials using Chebyshev nodal systems*

Jaime Díaz, Elías Berriochoa and Alicia Cachafeiro

Abstract

This piece of work is devoted to obtain formulae for Hermite (or Hermite type) interpolation problems on the bounded interval. The interpolation conditions gather the values of the polynomial and its first two derivatives at the nodal points and the nodal system is constituted by the Chebyshev or the extended Chebyshev points.

The basic idea is to translate each problem to a Hermite problem in the Laurent polynomial space using Joukowsky transformation. We can solve the problem in the space of Laurent polynomials by giving a explicit solution in terms of the natural basis of Laurent polynomials (see [2]). All the coefficients of the auxiliar problem can be computed in an easy and efficient way by means of the Fast Fourier Transform (FFT). Finally the solution of the original problem is obtained using Joukowsky transformation and identifying the coefficients properly; the final solution is so given in explicit form using the Chebyshev polynomials of first kind ($\{T_n(x)\}$) and using only FFT of sines and cosines.

Keywords: Hermite interpolation, Chebyshev points, extended Chebyshev points, Chebyshev-Lobatto nodes.

AMS Classification: 41A05, 42A15, 65D05, 42C05.

Bibliography

[1] **E. Berriochoa and A. Cachafeiro**, Algorithms for solving Hermite interpolation problems using the fast Fourier transform, *J. Comput. Appl. Math* **235** (2010) 882-894.

[2] **E. Berriochoa, A. Cachafeiro and J. Díaz**, Hermite Interpolation on the Unit Circle Considering up to the Second Derivative, *ISRN Mathematical Analysis*, vol. **2014** (2014) Article ID 808519, 10 pages, doi:10.1155/2014/808519.

*The research was supported by Ministerio de Ciencia e Innovación under grant number MTM2011-22713.

[3] **E. Berriochoa, A. Cachafeiro, J. Díaz and J. R. Illán**, Explicit formulas for Hermite type interpolation on the circle and applications (submitted).

[4] **E. Berriochoa, A. Cachafeiro, J. Díaz and J. R. Illán**, Algorithms and convergence for Hermite interpolation based on extended Chebyshev nodal systems, *Appl. Math. Comput.* **234** (2014) 223-236.

[5] **E. Berriochoa, A. Cachafeiro, J. Díaz and E. Martínez**, Rate of convergence of Hermite-Fejér interpolation on the unit circle, *J. Appl. Math.* **2013** (2013), Article ID 407128.

[6] **A. Poghosyan**, Fourier formulae for equidistant Hermite trigonometric interpolation, *Asian-European Journal of Mathematics* **4** (1) (2011) 127–144.

Jaime Díaz,
Departamento de Matemática Aplicada I,
Escuela de Ingeniería Industrial,
Universidad de Vigo,
36310 Vigo, Spain.
`jdiaz(at)uvigo.es`

Elías Berriochoa,
Departamento de Matemática Aplicada I,
Facultad de Ciencias,
Universidad de Vigo,
32004 Ourense, Spain.
`eberriochoa(at)uvigo.es`

Alicia Cachafeiro,
Departamento de Matemática Aplicada I,
Escuela de Ingeniería Industrial,
Universidad de Vigo,
36310 Vigo, Spain.
`acachafe(at)uvigo.es`

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

The electrostatic properties of zeros of exceptional Laguerre and Jacobi polynomials and stable interpolation*

Á. P. Horváth

Abstract

At first X_1-Jacobi and X_1-Laguerre polynomials, as exceptional orthogonal polynomial families were introduced by D. Gómez-Ullate, N. Kamran and R. Milson. The relationship between exceptional orthogonal polynomials and the Darboux transform is observed by C. Quesne. Higher-codimensional families were introduced by S. Odake and R. Sasaki. The properties of zeros of exceptional polynomials were investigated by D. Gómez-Ullate, F. Marcellán, R. Milson, D. Dimitrov and Yen Chi Lun.

We will examine the electrostatic properties of exceptional and regular zeros of X_m-Laguerre and X_m-Jacobi polynomials. Since there is a close connection between the electrostatic properties of the zeros and the stability of interpolation on the system of zeros, we can deduce some Egerváry-Turán type results as well.

Keywords: exceptional polynomials, zeros, minimal energy.
AMS Classification: 33E30,41A05.

Bibliography

[1] **D. K. Dimitrov and Yen Chi Lun**, Monotonicity, interlacing and electrostatic interpretation of zeros of exceptional Jacobi polynomials, *J. of Approx. Theory* **181** (2014) 18–29.

[2] **D. Gómez-Ullate, N. Kamran and R. Milson**, Exceptional orthogonal polynomials and the Darboux transformation, *J. Phys. A: Math. Theor.* **43** (2010) 434016 (16pp).

[3] **D. Gómez-Ullate, F. Marcellán and R. Milson**, Asymptotic and interlacing properties of zeros of exceptional Jacobi and Laguerre polynomials, *J. Math. Anal. Appl.* **399** (2013) 480–495.

*Supported by Hungarian National Foundation for Scientific Research, Grant No. K-100461

[4] **Á. P. Horváth**, Weighted Fejér Constants and Fekete Sets, *Acta Math. Hungar.* **141** (4), (2013) 366–382.

Á. P. Horváth,
Department of Analysis,
Budapest University of Technology and Economics,
1111 Budapest Egry J. u. 1. Hungary.
`g.horvath.agota@renyi.mta.hu`

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

Discrete orthogonal polynomials and parameter extraction in MOSFETs transitors*

D. Barrera, M. J. Ibáñez, A. M. Roldán, J. B. Roldán and R. Yáñez

Abstract

Transistors, and in particular MOSFETs (Metal Oxide Semiconductor Field Effect Transistors), are the most used basic building blocks of integrated circuits (ICs) [1]. The complexity of current chips makes essential their accurate characterization to use them for circuit design purposes. For each generation of transistors the main electrical features have to be modeled in order to reproduce them as a function of the voltages differences applied between their terminals. The models consist of a set of analytical equations and a set of parameters to include in those equations. A different set of parameters is used for each fabrication technology. These models are used in TCAD circuit simulation tools and also for hand-calculations used at the first stages of circuit design.

The extraction of the parameters of new technologies is essential since the capacities of circuit designers are dependant on the accuracy of model parameters that in many cases are linked to important physical effects.

Each parameter is obtained in a different way. However, few of them share some features in common, at least from the numerical viewpoint. In this respect, several parameters are obtained by means of extrapolation methods (for example threshold voltage calculation [1]), linear regression (determination of the body factor [1]), slope calculations (extraction of the DIBL parameter[1]), etc. In all these procedures, the determination of portions of curves that can be approximated by a straight line is crucial. In this work we just deal with this issue trying to shed light by means of advanced numerical techniques.

We have developed a method to determine the number of straight line portions contained in a curve in an automatic manner. The algorithm developed, based on discrete orthogonal polynomials, can be used for parameter extraction purposes. It consist on the isolation of straight line portions in experimental or simulated data and the determination of the slope of those curve sections to calculate one or more parameters of a compact model.

Keywords: discrete orthogonal polynomials, straight line portion, MOSFET.
AMS Classification: 33C45, 42C10, 65D10, 82D37.

*Junta de Andalucía (Project P08-TIC-3580) and Spanish Government (Project MTM2011-26468).

Bibliography

[1]**N. Arora**, *MOSFET modelling for VLSI simulation. Theory and practice*, reprinted by World Scientific, 2007.

[2] **A. F. Nikiforov and V.B. Uvarov**, *Special Functions of Mathematical Physics*, Birkhäuser Verlag, Basilea, 1988.

[3] **A.F. Nikiforov, S.K. Suslov and V.B. Uvarov**, *Classical Orthogonal Polynomials of a Discrete Variable*, Springer-Verlag, Berlin, 1991.

[4] **R. Álvarez-Nodarse**, *Polinomios hipergeométricos clásicos y q-polinomios*, Monografías del Seminario Matemático "García de Galdeano", Vol. 26, Prensas Universitarias de Zaragoza, Zaragoza, Spain, 2003.

D. Barrera, M. J. Ibáñez,
Department of Applied Mathematics,
University of Granada, 18071-Granada, Spain.
dbarrera@ugr.es, mibanez@ugr.es

A. M. Roldán, J. B. Roldán,
Department of Electronics,
University of Granada, 18071-Granada, Spain.
amroldan@ugr.es, jroldan@ugr.es

R. Yáñez,
Department of Applied Mathematics,
University of Granada, 18071-Granada, Spain.
ryanez@ugr.es

Inequalities for polynomials and rational functions normalized on an interval and circular arcs*

Sergei Kalmykov

Abstract

In this talk new covering and distortion theorems, and coefficient estimates for rational functions and algebraic polynomials with restrictions on circular arcs and on the the interval $[-1, 1]$ will be discussed.

Some of these theorems are consequences of the majorization principle [1] applied to an appropriate meromorphic function and dependent on Greenâs functions and inner radii of the domains complementary to circular arcs. Other results were proved using facts from the theory of univalent functions. In particular, the following theorem is true [2].

Theorem 1. *For a complex polynomial* $P(z) = c_n z^n + ... + c_k z^k, \quad n - k \geq 3, \; c_n c_k \neq 0$ *the following inequality*

$$\frac{4|c_n c_k| \sin^{2(n-k)}(\alpha/2)}{M^2 - m^2} \left(1 + \frac{1}{\sin(\alpha/2)} \left| \left(\frac{c_{n-1}}{2c_n} + \frac{\overline{c_{k+1}}}{2\overline{c_k}}\right) + (n-k)\cos^2(\alpha/2) \right| \right) \leq 1 \tag{1}$$

holds, where

$$m = \min_{-\alpha<\varphi<\alpha} |P(e^{i\varphi})|, \quad M = \max_{-\alpha<\varphi<\alpha} |P(e^{i\varphi})|.$$

Equality in (1) *is attained for*

$$P(z) = \begin{cases} \prod_{k=1}^{n/2} (z^2 - 2a_k z + 1), & \text{for even } n, \\ (z-1) \prod_{k=1}^{(n-1)/2} (z^2 - 2a_k z + 1), & \text{for odd } n, \end{cases}$$

where $a_k = \cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2} \cos \frac{\pi(2k-1)}{n}$.

*University of Szeged, Hungary; Far Eastern Federal University, Russia.

All mentioned statements are sharp and supplement recent results of Lukashov, Tyskevich, Maergoiz, Rybakova, Olesov, Dubinin and the speaker, as well.

Also we discuss some extremal properties of analogues for Chebyshev and Videnskii polynomials in rational spaces (for some details see [3, p. 139-145] and [4]).

Keywords: inequalities for polynomials and rational functions, Chebyshev polynomials, univalent functions, majorization principles.

AMS Classification: 41A17, 30A10.

Bibliography

[1] **V. N. Dubinin and S. I. Kalmykov**, A majoration principle for meromorphic functions, *Sbornik Mathematics* **198: 11-12**, (2007), 1737-1745.

[2] **S.I. Kalmykov**, On polynomials and rational functions normalized on the circular arcs, *Zap. Nauchn. Sem. POMI* **418** (2013) 105-120.

[3] **P. Borwein and T. Erdélyi**, *Polynomials and polynomial inequalities*, Springer-Verlag, New York, 1995.

[4] **A. L. Lukashov and S. V. Tyshkevich**, Extremal rational functions on several arcs of the unit circle with zeros on these arcs, *Izv. Saratov. Univ. Mat. Mekh. Inform.* **9:1** (2009), 8-13.

Sergei Kalmykov,
Bolyai Institute,
University of Szeged,
Szeged, Aradi v. tere 1, 6720, Hungary
&
Far Eastern Federal University,
8 Sukhanova Street,
Vladivostok, 690950, Russia.
sergeykalmykov@inbox.ru

Dual bases in subspaces

Scott N. Kersey

Abstract In this paper we study dual bases functions in subspaces. These are bases which are dual to subsets of functionals from larger linear spaces. Our goal is construct and derive properties of certain bases obtained from the construction, with primary focus on polynomial spaces in B-form. When they exist, our bases are always affine (not convex), and we define a symmetric configuration that converges to Lagrange polynomial bases. Because of affineness of our bases and linear polynomial reproduction, we are able to derive certain approximation theoretic results involving quasi-interpolation and a Bernstein-type operator. We also apply our construction to splines and multivariate polynomials. In particular, we give characterizations in the multivariate setting in B-form for existence of dual bases.

Keywords: dual bases functions, polynomials, splines.
AMS Classification: 41A05, 41A10, 41A15, 65D05, 65D07.

Bibliography

[1] **S. Kersey**, Dual basis in Subspaces of Inner Product Spaces *Applied Mathematics and Computation* **219(19)** (2013) 10012–10024.

[2] **S. Kersey**, Dual basis Functions in Subspaces, Manuscript (2014).

[3] **S. Kersey**, Dual basis Functions in Subspacs of Multivariate Polynomial Spaces, Manuscript (2014).

Scott Kersey,
Department of Mathematical Sciences,
Georgia Southern University, USA.
scott.kersey@gmail.com

About efficiency of approximation by ridge functions

Vitaly E. Maiorov

Abstract

The function of the form $r(a \cdot x)$ on $\mathbb{R}^d$ is calling ridge function with the direction a. We consider the approximation properties of variety $\mathcal{R}(A) = \{\sum_{a \in A} f(a \cdot x)\}$ formed by linear combination of ridge functions with directions from a finite set A.

The wide classes of functions (for example, harmonic functions) may be effectively approximated by $R(A)$ with adaptive choice of the direction set A comparatively to a fixed choice.

We show the connection between approximation by ridge functions and by trigonometric polynomials with floating harmonics. These results are apply to effective approximation of wide classes of smoothness functions by $R(A)$ with adaptive choice of the set A.

Vitaly E. Maiorov,
Alfréd Rényi Institute of Mathematics,
Hungarian Academy of Sciences,
H-1053 Budapest, Reáltanoda u. 13-15.
szabados.jozsef@renyi.mta.hu

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

Asymptotics of a family of varying discrete Sobolev orthogonal polynomials*

Juan F. Mañas-Mañas, Francisco Marcellán and Juan J. Moreno-Balcázar

Abstract

In this talk we consider the following inner product involving the Laguerre weight,

$$(f,g)_n = \frac{1}{\Gamma(\alpha+1)}\int_0^\infty f(x)g(x)x^\alpha e^{-x}dx + M_n f^{(j)}(0)g^{(j)}(0), \quad \alpha > -1, \tag{1}$$

where $\{M_n\}_n$ is a sequence of nonnegative numbers satisfying $M_n \sim Mn^\beta$ with $M > 0$ and $\beta \in \mathbb{R}$.

Asymptotic properties of the corresponding orthogonal polynomials with respect to (1) and of their zeros are obtained. In fact, we are interested in Mehler-Heine type formulae because they describe in detail the asymptotic differences between these Sobolev orthogonal polynomials and the classical Laguerre polynomials. In addition, we generalize some results appeared in the literature (for example, in [1]).

This work has been published in [2].

Keywords: Sobolev orthogonal polynomials, Mehler-Heine formulae.
AMS Classification: 33C47, 42C05.

Bibliography

[1] **H. Dueñas and F. Marcellán**, *The Laguerre–Sobolev–Type Orthogonal Polynomials*, J. Approx. Theory **162** (2010) 421–440.

[2] **J. F. Mañas Mañas, F. Marcellán and J. J. Moreno–Balcázar**, *Varying discrete Laguerre-Sobolev orthogonal polynomials: asymptotic behavior and zeros*, Appl. Math. Comput. **222** (2013) 612–618.

*The author FM is partially supported by Dirección General de Investigación, Ministerio de Economía y Competitividad Innovación of Spain, Grant MTM2012-36732-C03-01. The author JJMB is partially supported by Dirección General de Investigación, Ministerio de Ciencia e Innovación of Spain and European Regional Development Found, Grant MTM2011-28952- C02-01, and Junta de Andalucía, Research Group FQM-0229 (belonging to Campus of International Excellence CEI-MAR), and project P11-FQM-7276.

J.F. Mañas,
Departamento de Matemáticas,
Universidad Carlos III de Madrid.
jmanas@math.uc3m.es

F. Marcellán,
Departamento de Matemáticas,
Universidad Carlos III de Madrid.
pacomarc@ing.uc3m.es

J.J. Moreno-Balcázar,
Departamento de Matemáticas,
Universidad de Almería.
balcazar@ual.es

Sufficient conditions for sampling and interpolation on the sphere*

J. Marzo and B. Pridhnani

Abstract

The classical Marcinkiewicz-Zygmund inequality [1] states that, for $1 < p < \infty$, there exist constants $C_p > 0$ such that for any $P \in \mathcal{P}_n$

$$\frac{C_p^{-1}}{2n+1}\sum_{k=0}^{2n}|P(\omega_{kn})|^p \leq \int_0^{2\pi}|P(e^{i\theta})|^p d\theta \leq \frac{C_p}{2n+1}\sum_{k=0}^{2n}|P(\omega_{kn})|^p,$$

where $\mathcal{P}_n$ stands for the space of trigonometric polynomials of degree at most n, $\omega_{kn} = e^{\frac{2\pi i k}{2n+1}}$ are the $(2n+1)$th roots of unity, and the constants C_p are independent of the degree n.

This result can be rephrased as saying that the array of roots of unity is both sampling and interpolating for the spaces of trigonometric polynomials with the L^p norm.

I will talk about the generalization of these concepts to the sphere $\mathbb{S}^d$, $d \geq 2$, and its relation with "well distributed" points on the sphere. Finally, I will present our recent work about sufficient conditions for sampling and interpolation.

Keywords: Marcinkiewicz-Zygmund inequalities, interpolation, Laplace-Beltrami operator, points on the sphere.

AMS Classification: 65D32, 33C55, 65T40.

Bibliography

[1] **J. Marcinkiewicz and A. Zygmund**, Mean values of trigonometrical polynomials, *Fund. Math.* **28** (1937) 131–166.

J. Marzo,
Dept. Matemàtica Aplicada i Anàlisi,

*Supported by grants MTM2008-05561-C02-01 and 2009 SGR 1303.

Universitat de Barcelona,
Gran Via 585, 08007 Barcelona, Spain.
jmarzo@ub.edu

B. Pridhnani,
Dept. Matemàtica Aplicada i Anàlisi,
Universitat de Barcelona,
Gran Via 585, 08007 Barcelona, Spain.
bharti.pridhnani@ub.edu

Some remarks on Lagrange interpolation in weighted L^p−norm

Giuseppe Mastroianni

Abstract

The polynomial interpolation is an important tool in approximation theory, quadrature, numerical differentiation, as well as in the numerical treatment of functional equations.

In this talk we will show some results, obtained in the last years, concerning the behaviour of the Lagrange operator in different weighted spaces of functions. We will show that, under proper necessary and sufficient conditions, the Lagrange polynomials converge with the order of the best polynomial approximation in the considered function spaces.

Keywords: Lagrange interpolation, approximation by polynomials, orthogonal polynomials.

AMS Classification: 41A05, 41A10.

Giuseppe Mastroianni,
Department of Mathematics,
Computer Sciences and Economics,
University of Basilicata,
viale dell'Ateneo Lucano 10, 85100 Potenza, Italy.
`giuseppe.mastroiannni@unibas.it`

Dimension elevation vs. approximation by Bernstein operators

Marie-Laurence Mazure and Rachid Ait-Haddou

Abstract

Let us start with an infinite nested sequence of Extended Chebyshev spaces, each of them being assumed to possess a Bernstein basis. This situation automatically generates an infinite so-called *dimension elevation algorithm*. The question of convergence of the sequences of control polygons associated with given curves naturally arises.

On the other hand this situation also automatically generates an infinite sequence of *Bernstein operators* all reproducing the same two-dimensional space. Again, the question of convergence of this sequence towards identity naturally arises.

In this talk we examine the links existing between the two convergences, with still open questions.

Keywords: extended Chebyshev spaces, dimension elevation, blossoms, Bernstein operators.

AMS Classification: 41A36, 41A50, 65D17.

Bibliography

[1] **R. Ait-Haddou**, Dimension elevation in Müntz spaces: A new emergence of the Müntz condition, *J. Approx. Theory* **181** (2014) 6–17.

[2] **R. Ait-Haddou and M.-L. Mazure**, Approximation by Müntz spaces on positive intervals, *CRAS* **351** (2013) 849–852.

[3] **M.-L. Mazure**, Bernstein-type operators in Chebyshev spaces, *Num. Algorithms* **52** (2009) 93–128.

[4] **M.-L. Mazure**, Finding all systems of weight functions associated with a given Extended Chebyshev space, *J. Approx. Theory* **163** (2011) 363–376.

Marie-Laurence Mazure,
Université Joseph Fourier,
BP53, 38041 Grenoble Cedex 9, France.
`Marie-Laurence.Mazure@imag.fr`

Rachid-Ait-Haddou,
Geometric Modeling and Scientific Visualization Center,
KAUST, Saudia-Arabia.
`Rachid.AitHaddou@kaust.edu.sa`

Wiener type theorems for Jacobi series with nonnegative coefficients

Hrushikesh Mhaskar

Abstract

A well-known theorem by Norbert Wiener states that if f is a 2π–periodic function in $L^1(-\pi,\pi)$ with non-negative Fourier coefficients $c_n(f) \geq 0$ and $f \in L^2(-\delta,\delta)$ for some $\delta > 0$, then $f \in L^2(-\pi,\pi)$.

The goal of this paper is to describe some analogues of this theorem in the case of functions with nonnegative coefficients in their Jacobi expansions. We show that the L^p-integrability of such functions (with respect to the Jacobi weight) on an interval near 1 implies the L^p-integrability on the whole interval if p is an even integer. The Jacobi expansion of a function locally in L^∞ near 1 is shown to converge uniformly and absolutely on $[-1,1]$; in particular, such a function is shown to be continuous on $[-1,1]$. Similar results are obtained for functions in local Besov approximation spaces. This is joint work with S. Tikhonov.

H. N. Mhaskar,
California Institute of Technology, Pasadena,
Claremont Graduate University, Claremont.
`hmhaska@gmail.com`

Some results on Bernstein type inequalities in integral norms

Béla Nagy and Tamás Varga

Abstract

Bernstein inequality for polynomials on the inteval $[-1, 1]$ was established in 1912 and since then it has found many applications. In the last decades, this inequality was extended to wider class of sets, using potential theory. In particular, on the real line the sharp Bernstein type inequality was proved independently by Baran and Totik.

This inequality was extended to L^α norms where $1 \leq \alpha < \infty$ on union of finite intervals by Nagy and Toókos in 2013. We present some new partial results which are continuation of this research: extension to $0 < \alpha < 1$ and further information about the error term.

This is a joint work with Tamás Varga.

This research was realized in the frames of TÁMOP 4.2.4. A/2-11-1-2012-0001 ”National Excellence Program Elaborating and operating an inland student and researcher personal support system”. The project was subsidized by the European Union and co-financed by the European Social Fund.

Keywords: polynomial inequalities, Bernstein inequalities, potential theory, equilibrium measure.

AMS Classification: 41A17, 31A15.

Bibliography

[1] **M. Baran**, Complex equilibrium measure and Bernstein type theorems for compact sets in $\mathbb{R}^n$, *Proc. Amer. Math. Soc.* **123** (1995) 485–494.

[2] **B. Nagy and F. Toókos**, Bernstein inequality in L^α norms, *Acta Sci. Math. (Szeged)* **79** (2013) 139–174.

[3] **V. Totik**, Polynomial inverse images and polynomial inequalities, *Acta Math.* **187** (2001) 139–160.

Béla Nagy,
Bolyai Institute,
MTA-SZTE Analysis and Stochatics Research Group,
Szeged, H-6720, Aradi vértanúk tere 1, Hungary.
nbela@math.u-szeged.hu

Tamás Varga,
Bolyai Institute, University of Szeged,
Szeged, H-6720, Aradi vértanúk tere 1, Hungary.
vargata@math.u-szeged.hu

A remark on a conjecture of P. Nevai*

Incoronata Notarangelo

Abstract

In [4] P. Nevai conjectured that the analogous of a theorem in [3] could hold in the case in which the orthogonal polynomials $p_n(\alpha)$ are replaced by their derivatives. Moreover, in the same paper he proved the conjecture for generalized Jacobi weights.

Conjecture 1 (P. Nevai [4]). *Let $0 < p \leq \infty$. Then there is a constant $\mathcal{C}$ with the property that for every measure α supported in $[-1,1]$ such that $\alpha' > 0$ almost everywhere there, the inequality*

$$\left(\int_{-1}^{1}\left|\frac{f(t)}{\sqrt{\alpha'(t)}(1-t^2)^{3/4}}\right|^p \mathrm{d}t\right)^{1/p} \leq \mathcal{C}\liminf_{n\to\infty}\frac{1}{n}\left(\int_{-1}^{1}|f(t)p_n'(\alpha,t)|^p\,\mathrm{d}t\right)^{1/p}$$

holds for every measurable function f in $[-1,1]$.

Here, using some results in [1, 2], we show that the conjecture holds for doubling weights as well.

Keywords: orthogonal polynomials, derivatives of orthogonal polynomials, doubling weights.
AMS Classification: 42C05.

Bibliography

[1] **G. Mastroianni and V. Totik**, Weighted polynomial inequalities with doubling and A_∞ weights, *Constr. Approx.* **16** (2000), no. 1, 37–71.

[2] **G. Mastroianni and V. Totik**, Uniform spacing of zeros of orthogonal polynomials, *Constr. Approx.* **32** (2010), no. 2, 181–192.

*This work was supported by University of Basilicata (local funds) and by "National Group of Computing Science (GNCS–INDAM)": Progetto Giovani Ricercatori 2013 "Weighted polynomial approximation and its application to the numerical treatment of integral equations".

[3] **A. Máté, P. Nevai and V. Totik**, Necessary conditions for weighted mean convergence of Fourier series in orthogonal polynomials, *J. Approx. Theory* **46** (1986), no. 3, 314–322.

[4] **P. Nevai**, Orthogonal polynomials, recurrences, Jacobi matrices, and measures, in: *Progress in approximation theory*, A. A. Gonchar and E. B. Saff (ed.), Springer Ser. Comput. Math. 19, Springer, New York, 1992, 79–104.

[5] **I. Notarangelo**, On a conjecture of P. Nevai, *Publication de l'Institut Mathématique* (Beograd), to appear.

Incoronata Notarangelo,
Department of Mathematics,
Computer Sciences and Economics,
University of Basilicata,
viale dell'Ateneo Lucano 10, 85100 Potenza, Italy.
incoronata.notarangelo@unibas.it

Asymptotic expansions of multi-dimensional Mellin convolution integrals

Pedro J. Pagola and José Luis López

Abstract

In a recent paper [Lopez, 2008], we have presented a new, very general and simple method for deriving asymptotic expansions of

$$\int_0^\infty f(t)h(x\,t)dt$$

for small x. In this paper we generalize that method to multi-dimensional integrals of the form

$$\int_{\mathbb{R}_+^p} f(t_1,\dots,t_p)h(x\,t_1,\dots,x\,t_p)\,d^pt.$$

In this multi-dimensional case we require for f an extra condition not required in the one-dimensional case, namely, f must satisfy a certain homogeneity property in its variables. Watson's Lemma for multiple integrals is obtained as a corollary. An asymptotic expansion of the Lauricella's function F_A for large values of its variables is given as an illustration. The possible application of the method to study the asymptotic behavior near the origin of double Mellin-Barnes type convolution integrals [Yakubovich, 1993] is also indicated.

Keywords: asymptotic expansions of integrals, Mellin convolution integrals, Mellin transforms, Watson's lemma for multiple integrals, Lauricella's function F_A.
AMS Classification: 41A60, 30B40, 46F10.

Bibliography

[1] **J.L. López**, Asymptotic expansions of Mellin convolution integrals, *SIAM Rev* **50** (2) (2008) 275–293.

[2] **S. B. Yakubovich and S. L. Kalla**, On a new approach to convolution constructions, *Internat. J. Math. Math. Sci.* **16** 3 (1993) 435–448.

Pedro J.Pagola,
Departamento de Ingeniería Matemática e Informática,
Universidad Pública de Navarra.
`pedro.pagola@unavarra.es`

José Luis López,
Departamento de Ingeniería Matemática e Informática,
Universidad Pública de Navarra.
`jl.lopez@unavarra.es`

Bivariate orthogonal polynomials and 2D Toda Lattices

Cleonice F. Bracciali and Teresa E. Pérez

Abstract

Oscillations of an infinite system of points joined by spring masses where the interaction is is an exponential function of the distance between two spring masses are described by the so–called *Toda equations* (1989). An explicit solution of the Toda lattice equations in one time variable can be deduced by using orthogonal polynomials associated to an exponential modification of a measure.

In this work, we explore the connections between an infinite system of points in $\mathbb{R}^2$ described by a bi–dimensional version of the Toda equations with the standard theory of orthogonal polynomials in two variables.

Keywords: two variable orthogonal polynomials, Toda Lattice.

AMS Classification: 42C05, 33C50.

Bibliography

[1] **C. F. Dunkl and Y. Xu**, Orthogonal polynomials of several variables, *Encyclopedia of Mathematics and its Applications 81.* Cambridge University Press, 2001.

[2] **M. E. H. Ismail**, Classical and quantum orthogonal polynomials in one variable, *Encyclopedia of Mathematics and its Applications 98.* Cambridge University Press, 2005.

[3] **F. Peherstorfer**, On Toda lattices and orthogonal polynomials, *J. Comput. Appl. Math.* 133 (2001), 519–534.

[4] **P. M. Santini, M. Nieszporski and A. Doliwa**, Integrable generalization of the Toda law to the square lattice, *Physical Review E* **70** (2004), 056615.

Cleonice Bracciali,
Departamento de Matemática Aplicada,
UNESP–Universidade Estadual Paulista,
15054–000 São José do Rio Preto, SP, Brazil.
`cleonice@ibilce.unesp.br`

Teresa E. Pérez,
Departamento de Matemática Aplicada,
Facultad de Ciencias. Universidad de Granada,
18071 Granada, Spain.
`tperez@ugr.es`

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

Generalized coherent pairs of measures*

J. Petronilho

Abstract

In this talk we present a survey of some results obtained in the last years around the concept of coherent pair of measures and its generalizations, in the framework of the theory of orthogonal polynomials.

Let $(P_n)_n$ and $(Q_n)_n$ be two monic orthogonal polynomial sequences (OPS) with respect to the measures $\mathrm{d}\mu$ and $\mathrm{d}\nu$, respectively. $(\mathrm{d}\mu, \mathrm{d}\nu)$ is called a coherent pair of measures if $(P_n)_n$ and $(Q_n)_n$ are linked by a linear differential structure relation of the type

$$Q_n(x) = \tfrac{1}{n+1} P'_{n+1}(x) + s_n P'_n(x) \ , \quad n = 0, 1, 2, \cdots$$

where $(s_n)_n$ is a sequence of nonzero real numbers. The concept of coherent pair of measures was introduced by A. Iserles, P. E. Koch, S. P. Nørsett, and J. M. Sanz-Serna [On polynomials orthogonal with respect to certain Sobolev inner products, J. Aprox. Theory, **65**(2) (1991) 151-175] as an useful tool in Approximation Theory to explore polynomials orthogonal with respect to the Sobolev inner product

$$\langle f, g\rangle_\lambda := \int_{\mathbb{R}} fg\,\mathrm{d}\mu + \lambda \int_{\mathbb{R}} f'g'\,\mathrm{d}\nu \ ,$$

and since then it has been extensively studied by several authors, both from the algebraic and the analytical points of view. We will speak about a generalization of the above concept, called $(M, N)-$coherence of order (m, k), by considering a linear differential structure relation such as

$$\sum_{i=0}^{N} r_{i,n}\, Q^{(m)}_{n-i+m}(x) = \sum_{i=0}^{M} s_{i,n} P^{(k)}_{n-i+k}(x) \ ,$$

(the orders of derivatives, k and m, being arbitrarily fixed nonnegative integer numbers, and $(r_{i,n})_n$ and $(s_{i,n})_n$ being sequences of numbers fulfilling some appropriate conditions), as well as a Sobolev inner product of the form

$$\langle f, g\rangle_{\lambda,r} := \int_{\mathbb{R}} fg\,\mathrm{d}\mu + \lambda \int_{\mathbb{R}} f^{(r)}g^{(r)}\,\mathrm{d}\nu \ .$$

*CMUC, Department of Mathematics, University of Coimbra.

From the algebraic view point, one of the most interesting properties fulfilled by these generalized coherent pairs is the fact that the linear functionals with respect to which $(P_n)_n$ and $(Q_n)_n$ are orthogonal are related by a rational transformation (in the distributional sense) and they belong to the semiclassical class whenever $m \neq k$. We will also discuss some questions in Approximation Theory involving the OPS with respect to the above Sobolev inner product $\langle \cdot, \cdot \rangle_{\lambda,r}$ under the assumption that $(\mathrm{d}\mu, \mathrm{d}\nu)$ forms an $(M, N)-$coherent pair of measures of appropriate order, showing how the theory can be used for efficient evaluation of Sobolev-Fourier coefficients.

Some extensions to the so-called discrete OPS will be mentioned.

Most results presented in this talk were obtained in collaboration, with F. Marcellán, R. Álvarez-Nodarse, M.N. de Jesus, N. Pinzón-Cortés, and R. Sevinik-Adıgüzel.

Keywords: orthogonal polynomials, inverse problems, generalized coherent pairs, Sobolev orthogonal polynomials.

Bibliography

[1] **R. Álvarez-Nodarse, J. Petronilho, N. C. Pinzón-Cortés and R. Sevinik-Adıgüzel**, On linearly related sequences of difference derivatives of discrete orthogonal polynomials. Submitted.

[2] **M. N. de Jesus, F. Marcellán, J. Petronilho and N.C. Pinzón-Cortés**, (M, N)-coherent pairs of order (m, k) and Sobolev orthogonal polynomials, *J. Comput. Appl. Math.* **256** (2014) 16-35.

[3] **M. N. de Jesus and J. Petronilho**, Sobolev orthogonal polynomials and $(M, N)-$coherent pairs of measures, *J. Comput. Appl. Math.* **237** (2013) 83-101.

[4] **M. N. de Jesus and J. Petronilho**, On linearly related sequences of derivatives of orthogonal polynomials, *J. Math. Anal. Appl.* **347** (2008) 482-492.

[5] **J. Petronilho**, On the linear functionals associated to linearly related sequences of orthogonal polynomials, *J. Math. Anal. Appl.* **315** (2006) 379-393.

[6] **F. Marcellán and J. Petronilho**, Orthogonal polynomials and coherent pairs: the classical case, *Indag. Mathem.* N.S. **6** (83) (1995) 287-307.

J. Petronilho,
Department of Mathematics,
University of Coimbra,
EC Santa Cruz, 3001-501 Coimbra, Portugal.
josep@mat.uc.pt

Orthogonal and non-orthogonal approaches in Fourier continuous transforms

Darío Ramos López and Andrei Martínez-Finkelshtein

Abstract

Fourier transforms are widely used in several branches of mathematics and physics. In Fourier optics, they are a necessary step to obtain the point spread function (PSF) or the optical transfer function (OTF), which contain all the relevant optical information of a system.

According to Fourier optics, the diffraction integral depending on a defocus parameter transforms the complex pupil function into the impulse-response function, via a Fourier-type 2D integral.

For its computation, numerical procedures can be used (for instance, the bi-dimensional fast Fourier transform, FFT). As an alternative, semi-analytical methods can be employed to represent the complex pupil function and with it, a derivation of closed expressions for the diffraction integral is made.

One possibility is the method based on the Zernike polynomials, which are orthogonal on the unit disk, making use of the so-called *extended Nijboer-Zernike theory* [1]. This method presents some inconveniences, such as the limitation to symmetrical systems.

A different approach has been recently proposed [2, 3], by using the Gaussian radial basis functions, which are not orthogonal, but this is not a practical limitation.

In this talk, both procedures are derived and compared in terms of complexity and their performance is tested with numerical experiments, showing a major improvement in the computational time in the case of the non-orthogonal approach.

Keywords: Fourier transforms, diffraction integral, Zernike polynomials, extended Nijboer-Zernike theory, radial basis functions.

AMS Classification: 42A38, 65R10, 65T99.

Bibliography

[1] **A.J.E.M Janssen, J.J.M. Braat and P. Dirksen**, On the computation of the Nijboer-Zernike aberration integral at arbitrary defocus. *Journal of Modern Optics* **51** (2004) 687–703.

[2] **D. Ramos-López, A. Martínez-Finkelshtein and D.R. Iskander**, Computational aspects of the through-focus characteristics of a human eye *To appear in the Journal of the Optical Society of America A (JOSA A)* (2014).

[3] **D.R. Iskander, A. Martínez-Finkelshtein and D. Ramos-López**, Computation of 2D Fourier transforms and diffraction integrals using Gaussian radial basis functions *Preprint* (2014).

Darío Ramos López,
Edificio CITIC, Departamento de Matemáticas,
Universidad de Almería,
Ctra. Sacramento s/n, La Cañada de San Urbano,
04120, Almería (Spain).
dariorl@gmail.com

Andrei Martínez-Finkelshtein,
Edificio CITE III, Departamento de Matemáticas,
Universidad de Almería,
Ctra. Sacramento s/n, La Cañada de San Urbano,
04120, Almería (Spain).
andrei@ual.es

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

Characterization theorem for Laguerre-Hahn orthogonal polynomials on non-uniform lattices

Maria das Neves Rebocho

Abstract

Laguerre-Hahn orthogonal polynomials on non-uniform lattices were introduced by A.P. Magnus in [2]: a sequence of orthogonal polynomials is said to be Laguerre-Hahn if the corresponding formal Stieltjes function, S, satisfies a Riccati equation with polynomial coefficients

$$A(x)(\mathbb{D}S)(x) = B(x)(\mathbb{E}_1 S)(x)(\mathbb{E}_2 S)(x) + C(x)(\mathbb{M}S)(x) + D(x)\,, \quad A \neq 0\,, \tag{1}$$

where $\mathbb{D}$ is the divided difference operator involving the values of a function at two points, with the fundamental property that $\mathbb{D}$ leaves a polynomial of degree $n-1$ when applied to a polynomial of degree n [2, Eq. (1.1)]

$$(\mathbb{D}f)(x) = \frac{(\mathbb{E}_2 f)(x) - (\mathbb{E}_1 f)(x)}{y_2(x) - y_1(x)}\,, \tag{2}$$

with

$$(\mathbb{E}_1 f)(x) = f(y_1(x))\,, \;\; (\mathbb{E}_2 f)(x) = f(y_2(x))\,. \tag{3}$$

In this talk it is given a characterization theorem for Laguerre-Hahn orthogonal polynomials on non-uniform lattices [1]. The theorem proves the equivalence between the Riccati equation for the formal Stieltjes function, linear first-order difference relations for the orthogonal polynomials as well as for the associated polynomials of the first kind, and linear first-order difference relations for the functions of the second kind.

Keywords: orthogonal polynomials, non-uniform lattices, semiclassical linear functionals, structure relations.

AMS Classification: 33C45, 42C05.

Bibliography

[1] **A. Branquinho and M. N. Rebocho**, Characterization theorem for Laguerre-Hahn orthogonal polynomials on non-uniform lattices (submitted).

[2] **A.P. Magnus**, Associated Askey-Wilson polynomials as Laguerre-Hahn orthogonal polynomials, Springer Lect. Notes in Math. 1329, Springer, Berlin, 1988, pp. 261-278.

Maria das Neves Rebocho,
CMUC, Apartado 3008, EC Santa Cruz,
3001-501 Coimbra, Portugal
&
Department of Mathematics,
University of Beira Interior,
6201-001 Covilhã, Portugal.
`mneves@ubi.pt`

Convergence of rational Bernstein operators

Hermann Render

Abstract

In this talk we discuss convergence properties and error estimates of rational Bernstein operators introduced by P. Piţul and P. Sablonnière fixing linear polynomials. It is shown that the rational Bernstein operators converge to the identity operator if and only if the maximal difference between two consecutive nodes is converging to zero. Further a Voronovskaja theorem is given based on the explicit computation of higher order moments for the rational Bernstein operator.

Keywords: Rational approximants, Bernstein operator, positive operator.
AMS Classification: 41A20, 41A36.

Bibliography

[1] **P. Piţul and P. Sablonnière**, A family of univariate rational Bernstein operators, *J. Approx. Theory* **160** (2009) 39–55.

[2] **H. Render**, Convergence of rational Bernstein operators, *Appl. Math. Comput.* **232** (2014) 1076–1089.

[3] **I. Selimovic**, New bounds on the magnitude of the derivative of rational Bézier curves and surfaces, *Comput. Aided Geom. Design* **22** (2005) 321–326.

[4] **G.T. Tachev**, The complete asymptotic expansion for Bernstein operators, *J. Math. Anal. Appl.* **385** (2012), 1179–1183.

[5] **G.T. Tachev**, The rate of approximation by rational Bernstein functions in terms of second order moduli of continuity, *Numer. Funct. Anal. Optim.* **33** (2012), 206–215.

Hermann Render,
School of Mathematical Sciences,
University College Dublin,
Dublin 4, Ireland.
hermann.render@ucd.ie

Filling holes with smoothness conditions

M.A. Fortes, P. González, M. Pasadas and M.L. Rodríguez

Abstract

In this work we develop a method to fill a hole in a 3D scattered dataset, which may come from an explicit surface or have been obtained in a empirical way. We construct a surface which is very close to the given dataset and which fills the hole in such a way that the final reconstruction has the desired smoothness C^k, for $k \geq 0$. We give results which prove the existence and uniqueness of solution of the proposed method, and we present several examples which show the efficiency of the developed theory.

Keywords: Hole filling, Powell-Sabin splines, minimal energy surfaces.
AMS Classification: 41A15, 65D07, 65D10, 65D17.

Bibliography

[1] **D. Barrera, M. A. Fortes, P. González and M. Pasadas**, Minimal energy surfaces on Powell-Sabin type triangulations, *Applied Numerical Mathematics* 58(5) (2007), 635–645.

[2] **C. K. Chui and M-J. Lai**, Filling polygonal holes using C^1-cubic triangular patches, *Computer Aided Geometric Design* 17 (2000), 297–307.

[3] **M. J. D. Powell and M. A. Sabin**, Piecewise quadratic approximations on triangles, *ACM Trans. Math. Software* 3(4) (1977), 316–325.

M. A. Fortes,
Dept. Applied Mathematics, University of Granada,
Granada, Spain.
mafortes@ugr.es

P. González,
Dept. Applied Mathematics, University of Granada,

Granada, Spain.
prodelas@ugr.es

M. Pasadas,
Dept. Applied Mathematics, University of Granada,
Granda, Spain.
mpasadas@ugr.es

M. L. Rodríguez,
Dept. Applied Mathematics, University of Granada,
Granda, Spain.
miguelrg@ugr.es

Dynamics of equilibrium measures in the presence of certain rational external fields

J.F. Sánchez-Lara and R. Orive

Abstract

The subject of the present talk is the study of families of equilibrium measures in the real line in the presence of rational external fields (that is, when the derivative of the field is a rational function). It is well known that the support of an equilibrium measure in a real analytic external field is comprised of a finite number of intervals. In the last years, many papers have dealt with equilibrium problems in the presence of polynomial external fields, paying special attention to the evolution of the support of the equilibrium measure when the total mass of the measure (also regarded as the "time" or "temperature") varies.

In the present talk, we center in the case of rational fields with a part created by a number of attractive or repulsive charges placed in $\mathbb{C}$ and also study the dynamics when other parameters of the external field vary, as for example the position of the charges. These external fields are related for instance with generalized Heine-Stieltjes polynomials and are present on a number of physical problems related to random matrix models.

Keywords: logarithmic potentials.
AMS Classification: 31A15, 30E20, 30E25.

J.F. Sánchez-Lara,
Universidad de Granada.
jslara@ugr.es

R. Orive,
Universidad de La Laguna.
rorive@ull.es

Approximation by Chlodowsky type Baskakov operators

Sevilay Kırcı Serenbay, İbrahim Büyükyazıcı and Çiğdem Atakut

Abstract

In this study, we give a generalization of the Baskakov type operators , special case of this operator includes Chlodowsky type MKZ operator (introduced by L. Rempulska and M. Skorupka [8]), for functions on $[0, b_n]$ $(n \to \infty)$ extending infinity and prove some approximation properties of these operators with the help of a Korovkin type theorem. Secondly, we compute the rate of convergence of the operators by means of asymptotic inequality and also we introduce modify the operators for differentiable functions. Finally,by using the operators, we present an application to differential equation.

Keywords: approximation, Chlodowsky type MKZ operators, Baskakov operators, asymptotic approximation, differential equation.

AMS Classification: 41A25, 41A36.

Bibliography

[1] **J. A. H. Alkemade**, The second moment for the Meyer - König and Zeller operators, *J. Approx. Theory*, **40** (1984), 261-273.

[2] **F. Altamore and M. Campiti**, *Korovkin type approximation theory and its applications*, Berlin-New York, 1994.

[3] **K. Balazs**, Approximation by Bernstein type rational functions, *Acta Mathematica Academiae Scientiarum Hungaricae Tomus* **26**(1-2) (1975),123-134.

[4] **V.A. Baskakov**, On a construction of converging sequences of linear positive operators, *In: Studies of Modern Problems of Constructive Theory of Functions*, Fizmatgiz, Moskow (1961), 314-318, In Russian.

[5] **M. Becker and R. J. Nessel**, A Global Approximation Theorem for Meyer-König and Zeller Operators, *Math. Z.* **160** (1978), 195-206.

[6] **A. D. Gadjiev, R. O. Efendiev and E. İbikli**, Generalized Bernstein-Chlodowsky polynomials, *Rocky Mount. J. Math.* **28**(4) (1998), 1267-1277.

[7] **N. İspir and Ç. Atakut**, Approximation by generalized Balazs type rational functions, *Int. J. Comput. Numer. Anal.* **4**(3) (2003), 297-316.

[8] **L. Rempulska and M. Skorupka**, Approximation by generalized MKZ-operators in polynomial weighted spaces, *Anal. Theory Appl.* **23**(1) (2007), 64–75.

Sevilay Kırcı Serenbay,
Başkent University,
Department of Mathematics Education,
Ankara-Turkey.
kirci@baskent.edu.tr

İbrahim Büyükyazıcı,
Ankara University, Faculty of Science,
Department of Mathematics,
Tandogan-Ankara- Turkey.
ibuyukyazici@gmail.com

Çiğdem Atakut,
Ankara University, Faculty of Science,
Department of Mathematics,
Tandogan-Ankara- Turkey.
atakut@science.ankara.edu.tr

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

On a quasi-interpolating Bernstein operator

József Szabados

Abstract

We consider a special case of the modification of Lagrange interpolation due to Bernstein. Compared to Lagrange interpolation, these operators interpolate at less points, but they converge for all continuous functions in case of the Chebyshev nodes. Upper and lower estimates for the rate of convergence are given, and the saturation problem is partially solved.

Keywords: Bernstein operator, Lagrange interpolation, Chebyshev polynomial, error estimate, saturation.
AMS Classification: 41A25, 41A40.

Bibliography

[1] **S. N. Bernstein**, Sur une modification de la formule d'interpolation de Lagrange, *Kharkov Zap. Matem. Obs.* **(4) 5** (1932) 49–57.

[2] **Z. Ditzian and V. Totik**, *Moduli of Smoothness*, Springer, 1989.

[3] **L. Szili and P. Vértesi**, On a convergent process of S. Bernstein, *J. of Applied Math. and Computation*, to appear.

József Szabados,
Alfréd Rényi Institute of Mathematics,
Hungarian Academy of Sciences,
H-1053 Budapest, Reáltanoda u. 13-15.
szabados.jozsef@renyi.mta.hu

• V Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 23rd-26th, 2014

Construction of a non-linear quasi-interpolation based on the cubic B-splines

Hyoseon Yang and Jungho Yoon

Abstract

Quasi-interpolation is a very useful tool for multivariate data approximation. It enables very large-scale data sets to be handled efficiently. The linear quasi-interpolation has advantages in simplicity and fast computation, but often suffers from ringing artifacts when approximating across discontinuities. In this regard, for a better match to the local structures, this paper presents a non-linear quasi-interpolation method. To this end, we first discuss a smoothness indicator which measures the local smoothness of the given data and then construct explicitly a local non-linear approximation scheme to approximate data with singularities. Error analysis of the proposed scheme is provided by showing that the scheme has the same approximation order as the corresponding linear B-spline method. Finally, some numerical experiments are performed to demonstrate the ability of the new scheme to reduce the ringing artifacts near discontinuities.

Keywords: non-linear approximation, quasi-interpolation, cubic B-spline, approximation order, smoothness indicator.
AMS Classification: 41A05, 41A15, 41A25, 41A60.

Bibliography

[1] **F. Arandiga, A. Cohen, R. Donat and N. Dyn**, Interpolation and approximation of piecewise smooth functions, *SIAM J. Numer. Anal.* **43** (2005) 41–57.

[2] **F. Keinert**, *Wavelets and Multiwavelets*, Chapman & Hall/CRC, Boca Raton, 2000.

[3] **A. Levin**, Polynomial generation and Quasi-interpolation in stationary non-uniform subdivision, *Comput. Aided. Geom. Design* **20(1)** (2003) 41–60.

[4] **Y. Lipman and D. Levin**, Approximating piecewise smoothing functions *IMA J. of Numer. Anal.* **30 (4)** (2010) 1159-1183.

[5] **C.W. Shu**, Essentially non-oscillatory and weighted essentially non-oscillatory schemes for hyperbolic conserbation laws, *Advanced Numerical Approximation of Nonlinear Hyperbolic Equations, Lecture Notes in Mathematics* **1697** (1998) 325–432.

Hyoseon Yang,
Department of Mathematics,
Ewha W. University,
Seoul 120-750, Republic of Korea.
`hsyang1989@gmail.com`

Jungho Yoon,
Department of Mathematics,
Ewha W. University,
Seoul 120-750, Republic of Korea.
`yoon@ewha.ac.kr`

New ideas and results for the Gasca-Maeztu conjecture

Hakop Hakopian, Kurt Jetter and Georg Zimmermann

Abstract

An n-poised set in two dimensions is a set of nodes admitting unique bivariate interpolation with polynomials of total degree at most n. We are interested in poised sets with the property that all fundamental polynomials are products of linear factors. In 1982, M. Gasca and J. I. Maeztu conjectured that every such set necessarily contains $n+1$ collinear points. The case $n=4$ was proved for the first time in 1990 by J. R. Busch [1], later with different methods by J. M. Carnicer and M. Gasca [2], and later again with different methods by the authors [3]. The case $n=5$ was shown by the authors in [4]. We present new ideas pointing towards a general result.

Keywords: polynomial interpolation, Gasca-Maeztu conjecture, fundamental polynomial, maximal line, poised set.
AMS Classification: 41A28, 41A40, 41A60, 41A05, 41A63, 14H50.

Bibliography

[1] **J. R. Busch**, A note on Lagrange interpolation in $\mathbb{R}^2$, *Rev. Un. Mat. Argentina* **36** (1990) 33–38.

[2] **J. M. Carnicer and M. Gasca**, A conjecture on multivariate polynomial interpolation, *Rev. R. Acad. Cienc. Exactas Fís. Nat. (Esp.), Ser. A Mat.* **95** (2001) 145–153.

[3] **H. Hakopian, K. Jetter, and G. Zimmermann**, A new proof of the Gasca-Maeztu conjecture for $n=4$, *J. Approx. Theory* **159** (2009) 224–242.

[4] **H. Hakopian, K. Jetter, and G. Zimmermann**, The Gasca-Maeztu conjecture for $n=5$, *Numer. Math.* (to appear).

Hakop Hakopian,
Department of Informatics and Applied Mathematics,
Yerevan State University,
A. Manukyan St. 1,
0025 Yerevan, Armenia.
`hakop@ysu.am`

Kurt Jetter,
Georg Zimmermann,
Institut für Angewandte Mathematik und Statistik,
Universität Hohenheim,
70593 Stuttgart, Germany.
`Kurt.Jetter@uni-hohenheim.de`
`Georg.Zimmermann@uni-hohenheim.de`