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SHAPE PRESERVING APPROXIMATION OF PERIODIC FUNCTIONS

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Abstract

We wish to approximate a continuous 2π-periodic function that changes its monotonicity or convexity finitely many times in its period, by trigonometric polynomials that follow exactly these changes, namely are nondecreasing and nonincreasing, respectively convex or concave, exactly where the function is. Especially, we investigate the validity of Jackson-type estimates in this type of approximation. This type of approximation is called comonotone, respectively coconvex approximation. There are no Jackson type estimates for the so called co-q-monotone, $q > 2$, approximation. We discuss whether these estimates depend on the disposition of the points of change, namely the extremum, respectively, the inflection points of the function to be approximated. It is interesting to point out that the results for coconvex approximation are similar to those we know for coconvex algebraic polynomials approximation of a continuous function on a finite interval, while the results for comonotone approximation are substantially different than the analogous results for comonotone algebraic polynomials approximation of a continuous function on a finite interval.

Let $f \in C$ be a 2π-periodic function that changes its monotonicity, respectively convexity $2s$-times, $s \geq 1$, in a period at points Y_s. We wish to approximate f by trigonometric polynomials T_n of degree $< n$, which follow its changes in monotonicity, respectively convexity. For $l = 1, 2$, we denote by

$$E_n^{(l)}(f, Y_s) = \inf \|f - T_n\|,$$

where the infimum is taken on all such T_n, the degree of such approximation.

For $f \in C^r$, $r \geq 0$, we are interested in estimates of the form

$$E_n^{(l)}(f, Y_s) \leq \frac{c(r, k, s)}{n^r} \omega_k(f^{(r)}, 1/n), \quad n \geq N,$$

where ω_k is the kth modulus of smoothness of f.

In general, such estimates exist with $N = N(r, k, Y_s)$. We will determine for which triplets (r, k, s), we may take $N = 1$.

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