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Asymptotic behaviour of certain semigroups generated by differential operators

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Abstract

Let $(T(t))_{t\geq 0}$ be the semigroup on $C[0,1]$ generated by the differential operator $\frac{1}{2}x^a(1-x)^b d^2/dx^2$, $a, b \geq 1$. We obtain quantitative results concerning the behaviour of $T(t)f$ when $t \to 0$, respectively $t \to \infty$, and $f \in C[0,1]$.

Keywords: One-parameter semigroup, asymptotic behaviour.

MSC: Primary 47D06; Secondary 41A36.

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§1. Introduction

The differential operator

$$A := \frac{1}{2}x^a(1-x)^b\frac{d^2}{dx^2},\ a, b \geq 1,$$

with a suitable domain $D(A)$, was intensively investigated as infinitesimal generator of a strongly continuous semigroup $T(t))_{t\geq 0}$ on $C[0,1]$ with the uniform norm $\|\|$. For $a = b = 1$, it is related to the classical Bernstein operators; see Chapter 6 of [2].

More generally, perturbed versions of A on bounded or unbounded intervals, with suitable weighted norms, were considered in [1, 5, 7, 12, 15]; recent results in this direction can be found in [3, 4, 8, 9] and the references therein.