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Penalized Least Squares Approximation Problems

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Abstract

Let X, Y and S be linear spaces, $S \subseteq X \cap Y$. Let $\|\cdot\| : X \to \mathbb{R}$ be a semi-norm on X induced by a semi-definite inner product $\langle \cdot, \cdot \rangle$ in X, and let $|\cdot| : Y \to \mathbb{R}$ be a semi-norm on Y induced by a semi-definite inner product $[\cdot, \cdot]$ in Y. We study

Problem P. *Given $f \in X$, $g \in Y$ and $\lambda > 0$, find $s_\lambda := s_\lambda(f, g) \in S$ which satisfies*

$$\|f - s_\lambda\|^2 + \lambda \ |g - s_\lambda|^2 \ = \ \min_{s \in S} \Big(\|f - s\|^2 + \lambda \ |g - s|^2 \Big).$$

If S is a Hilbert space with respect to $\langle \cdot, \cdot \rangle + [\cdot, \cdot]$, then s_λ converges to some limit s_0 if $\lambda \to 0$. We characterize s_0 and find new estimates for $s_\lambda - s_0$. Schoenberg's natural splines and Tikhonov regularization are included as examples.

Keywords: Least squares approximation, penalized least squares approximation, convergence of penalized least squares approximations.

MSC: Primary 41A30, 41A65; Secondary 41A15, 41A28.

§1. Introduction

Let $\Delta : a = x_0 < x_1 < \cdots < x_n = b$, $m \in \mathbb{N}$, $m \le n + 1$, and $f \in C[a, b]$. Among all $u \in W_2^m(a, b)$ satisfying $u(x_j) = f(x_j)$, $j = 0, 1, \ldots, n$, there exists a unique function