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Nikolskii-type estimates for coconvex approximation of functions with one inflection point†

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Abstract

For each $r \in \mathbb{N}$ we prove the Nikolskii type pointwise estimate for coconvex approximation of functions $f \in W^r$, the subspace of all functions $f \in C[-1,1]$, possessing an absolutely continuous $(r-1)$st derivative on $(-1,1)$ and satisfying $f^{(r)} \in L_\infty[-1,1]$, that change their convexity once on $[-1,1]$.

Keywords: Coconvex polynomial approximation, pointwise estimates.

MSC: 41A10, 41A17,41A25, 41A29.

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§1. Introduction and main result

Denote by $\mathcal{P}_n$ the space of algebraic polynomials of degree $< n$, $n \in \mathbb{N}$. Set $\|\cdot\| := \|\cdot\|_{L_\infty[-1,1]}$, in particular $\|f\| = \|f\|_{C[-1,1]}$, if $f \in C[-1,1] =: C^0[-1,1]$. For $r \in \mathbb{N}$ let $C^r[-1,1]$ be the space of all r times continuously differentiable functions on $[-1,1]$, and let W^r be the subspace of all functions $f \in C[-1,1]$, possessing an absolutely continuous $(r-1)$st derivative on $(-1,1)$ and satisfying $f^{(r)} \in L_\infty[-1,1]$. Put

$$\varphi(x) := \sqrt{1-x^2}, \quad \varphi_n(x) := \frac{1}{n} + \varphi(x), \quad \text{and} \quad \rho_n(x) := \frac{1}{n}\varphi_n(x).$$

†Part of this work was done while the first and last authors visited Tel Aviv University.