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The Multiresolution Structure of Pairs of Dual Wavelet Frames for a Pair of Sobolev Spaces

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Abstract

We study the multiresolution structure of wavelet frames. It is known that the internal structure of almost any nontrivial overcomplete dyadic tight wavelet frame's underlying multiresolution analysis $(V_j)_{j\in\mathbb{Z}}$ is degenerated in $L_2(\mathbb{R})$. More precisely, the relation $W_0 \oplus V_0 = V_1$, that would hold for wavelet bases, collapses into $W_0 = V_1$, where W_0 is the closed linear span of the wavelets' integer shifts.

In the present paper, we extend the latter result in three ways: First and most significantly, we don't require a tight wavelet frame and verify that the result still holds for a pair of dual wavelet frames. Secondly, we allow for general scaling matrices. Thirdly, the pair of dual wavelet frames is not required to form a frame for $L_2(\mathbb{R}^d)$ but only for a pair of dual Sobolev spaces $(H^s(\mathbb{R}^d), H^{-s}(\mathbb{R}^d))$. Thus, the dual refinable function does not have to be in $L_2(\mathbb{R}^d)$. Finally, we construct pairs of dual wavelet frames for a pair of dual Sobolev spaces from any pair of multivariate refinable functions.

Keywords: Pairs of dual wavelet frames, multiresolution analysis, mixed extension principle, shift-invariant space, Sobolev space.

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