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Rational Gauss-Radau and rational Szegő-Lobatto quadrature on the interval and the unit circle respectively

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Abstract

We present a relation between rational Gauss-Radau quadrature formulas with one fixed node in the open interval $(-1,1)$ that approximate integrals of the form $J_\mu(f) = \int_{-1}^{1} f(x)d\mu(x)$ and rational Szegő-Lobatto quadrature formulas with two fixed nodes on the complex unit circle that approximate integrals of the form $I_{\mathring{\mu}}(f) = \int_{-\pi}^{\pi} f(e^{\mathbf{i}\theta})d\mathring{\mu}(\theta)$. The measures μ and $\mathring{\mu}$ are assumed to be positive bounded Borel measures on the interval $[-1,1]$ and the complex unit circle respectively, and are related by $\mathring{\mu}'(\theta) = \mu'(\cos\theta)\,|\sin\theta|$. Next, we derive convergence results for the rational Szegő-Lobatto quadrature rules and use the relation to obtain similar convergence results for rational Gauss-Radau quadrature rules. Further, we include some illustrative numerical examples.

Keywords: Rational Gauss-Radau quadrature, rational Szegő-Lobatto quadrature, convergence, quasi-orthogonal rational functions, para-orthogonal rational functions.

MSC: Primary 42C05; Secondary 65D32.

§1. Introduction

In this paper we shall explore the interplay between quadrature formulas for integrals of the form $J_\mu(f) = \int_{-1}^{1} f(x)d\mu(x)$, where μ is a positive Borel measure on $I = [-1,1]$, and