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# A six-dimensional counterexample for the $GM_d$ conjecture

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## Abstract

The conjecture  $GM_d$  by C. de Boor was based on a conjecture by M. Gasca and J.I. Maeztu about interpolation in the plane. This conjecture was formulated by M. Gasca and J. I. Maeztu to argue that the construction of Newton formula based on the system of interpolation nodes of Berzolari and Radon was more general and covered all examples of sets satisfying the geometric characterization described by Chung and Yao. Some examples and facts observed in planar examples seemed to suggest that the system of interpolation nodes of Berzolari and Radon could describe much more examples than the Chung and Yao construction and even that each  $GC$  set in any dimension could be described as a set of interpolation nodes obtained by a multidimensional generalization of Berzolari-Radon construction.

In this paper we provide a counterexample which shows that, at least in  $\mathbb{R}^6$ , there are  $GC$  sets that cannot be described as the set of interpolation nodes of Berzolari-Radon interpolation system.

**Keywords:** multivariate polynomial interpolation, geometric characterization, maximal hyperplane, fundamental polynomial,  $GC_n$  set, natural lattice.

**MSC:** Primary 41A05; Secondary 41A63.

## §1. Introduction

Let us start with some notation. Denote

$$\bar{x} = (x_1, x_2, \dots, x_d) \in \mathbb{R}^d, \quad \alpha = (\alpha_1, \alpha_2, \dots, \alpha_d) \in \mathbb{Z}_+^d,$$

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