



Affine generalised barycentric coordinates

Shayne Waldron

Abstract

For a given set of points in \mathbb{R}^d , there may be many ways to write a point x in their affine hull as an affine combination of them. We show that there is a *unique* way which minimises the sum of the squares of the coefficients. It turns out that these coefficients, which are given by a simple formula, are affine functions of x , and so generalise the barycentric coordinates. These *affine* generalised barycentric coordinates have many nice properties, e.g., they depend continuously on the points, and transform naturally under symmetries and affine transformations of the points. Because of this, they are well suited to representing polynomials on polytopes. We give a brief discussion of the corresponding Bernstein-Bézier form and potential applications, such as finite elements and orthogonal polynomials.

Keywords: barycentric coordinates, Wachspress coordinates, mean value coordinates, multivariate Bernstein polynomials, least squares method.

MSC: Primary 41A65, 65D17, 52B11, 41A10; Secondary 42C15, 41A36, 65D10.

§1. Introduction

A sequence p_1, \dots, p_n of $n = d + 1$ points in \mathbb{R}^d is affinely independent if and only if each point $x \in \mathbb{R}^d$ can be written uniquely as an *affine combination* of them, i.e.,

$$x = \sum_j \lambda_j(x) p_j, \quad \sum_j \lambda_j(x) = 1. \quad (1.1)$$

Communicated by

P. Sablonnière

Received

August 16, 2011

Accepted

September 28, 2011