ISSN: 1889-3066

Web site: jja.ujaen.es

Jaen J. Approx. 3(2) (2011), 261–270

Jaen Journal on Approximation

The loss of Hölder regularity of four-point interpolatory subdivision on irregularly spaced points

Michael S. Floater

Communicated by
K. Jetter

Received
August 31, 2011
Accepted
November 24, 2011

Abstract

Daubechies, Guskov and Sweldens studied four-point, cubic-based interpolatory subdivision on irregularly spaced grid points and showed that if a 'dyadic' mesh-ratio, λ, where $1/2 \leq \lambda \leq 1$, satisfies the bound $\lambda \leq 2/3$, the limit function has Hölder regularity $C^{2-\epsilon}$ for any small $\epsilon > 0$. They also conjectured that $C^{2-\epsilon}$ regularity is maintained for all $\lambda < 1$. We show, on the contrary, that for certain grids, with $\lambda > \lambda_1$, where $\lambda_1 \approx 0.8847$, regularity is lost. In fact, the regularity of the scheme can approach C^1 as λ approaches 1.

Keywords: interpolatory subdivision, polynomial interpolation, Hölder regularity.

MSC: Primary 65D05, 65D10; Secondary 26A16, 94A12.

§1. Introduction

Subdivision has become an important tool for curve and surface generation in computer-aided geometric design and computer graphics, as well as for signal and image processing, due to its use in constructing multiresolution analyses [1, 2, 3]. One of the best known schemes is the interpolatory four-point scheme based on cubic polynomial interpolation that was studied both by Dubuc [5] and, as a special case of a more general four-point