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# From Taylor interpolation to Hermite interpolation via duality

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## Abstract

The present work concerns  $W$ -spaces, that is, spaces which permit Taylor interpolation on a given interval. We introduce the critical length of any given  $W$ -space  $\mathbb{E}$  as the supremum of all positive  $h$  ensuring that  $\mathbb{E}$  permits Hermite interpolation (*i.e.*,  $\mathbb{E}$  is an Extended Chebyshev space) on any subinterval of length  $h$ . The critical length may be equal to 0, but it is always positive if the interval is closed and bounded. Any  $W$ -space is allocated to a dual space. When the dual space is a  $W$ -space in turn, we can take advantage of its presence to calculate the critical length.

The notion of critical length was first introduced in [3] for null spaces of linear differential operator with constant coefficients. As a special case, the use of duality gives new insights into the practical expressions to obtain the critical length of such null spaces.

**Keywords:** Taylor interpolation, Hermite interpolation,  $W$ -spaces, Extended Chebyshev spaces, dual  $W$ -spaces.

**MSC:** 41A05, 41A50, 65D07, 65D17.

## §1. Introduction

On a given non-trivial real interval  $I$ , a  $W$ -space is a space which permits Taylor interpolation, while an Extended Chebyshev space is a space which permits Hermite interpolation.

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