ISSN: 1889-3066
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Web site: jja.ujaen.es

Jaen J. Approx. 5(1) (2013), 1–18

Jaen Journal on Approximation

Moments of shifted radial functions with respect to orthogonal system of polynomials on the ball

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Communicated by M. Buhmann

Received December 12, 2011
Accepted February 11, 2013

Abstract

The moments $\langle g, P_I \rangle$ of shifted radial functions g with respect to some orthogonal polynomial system $\{P_I\}$ of functions on the unit ball are calculated. Also with the help of these moments we prove the following result: given a natural number n we denote by $\mathcal{P}_n^d$ the space of polynomials on $\mathbb{R}^d$ of total degree n and by $\mathcal{Q}_{2n}^d$ some space of polynomials such that $\mathcal{P}_n^d \subset \mathcal{Q}_{2n}^d \subset \mathcal{P}_{2n}^d$. Let $\mathcal{H}_n$ be the space of harmonic polynomials of degree n and N_n be the dimension of $\mathcal{H}_n$. Then every polynomial p in the space $\mathcal{Q}_{2n}^d$ may be represented by a linear combination of N_n shifted radial functions of the form $g_k(\|x + a_k\|)$, $g_k \in \mathcal{P}_{2n}^1$, $a_k \in \mathbb{R}^d$, $k = 1, ..., N_n$, if and only if the set $\{a_1, ..., a_{N_n}\}$ is a uniqueness set for the space $\mathcal{H}_n$.

Keywords: radial functions, moments, approximation, harmonic analysis.

MSC: 41A05, 41A30, 65D05, 42B05.

§1. Introduction and main results

Let $\mathbb{R}^d$ be the real Euclidean space with norm $\|x\| = (\sum_{i=1}^d x_i^2)^{1/2}$. Let $s = (s_1, ..., s_d)$ be any vector with nonnegative integer coordinates. We denote $x^s = x_1^{s_1} \cdots x_d^{s_d}$ and $|s| = s_1 + \cdots + s_d$. Consider the space $\mathcal{P}^d = \text{span}\{x^s : s \in \mathbb{Z}_+^d\}$ of all polynomials on