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Pointwise estimates of coconvex approximation

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Abstract

Let $f \in C[-1,1]$ change its convexity in the interval. We are interested in estimating the degree of approximation of f by polynomials which are coconvex with it, namely, polynomials that change their convexity exactly at the points where f does. We obtain Nikolskii-type pointwise estimates for such constrained approximation.

Keywords: coconvex polynomial approximation, pointwise estimates.

MSC: 41A10, 41A17, 41A25, 41A29.

§1. Introduction and main results

Let $f \in C[-1,1]$ change its convexity finitely many, say $s \geq 0$, times in the interval. We are interested in pointwise estimates on the approximation of f by algebraic polynomials that are coconvex with it, that is, that change their convexity exactly at the points where f does. Specifically, denote by $\mathbb{Y}_s$, $s \in \mathbb{N}$, the set of all collections $Y_s := \{y_i\}_{i=1}^s$, such that $-1 < y_s < \cdots < y_1 < 1$. Let $\Delta^{(2)}(Y_s)$ denote the collection of all functions $f \in C[-1,1]$ that change convexity at the set Y_s and are convex in $[y_1, 1]$. Namely, in the interval $[y_{i+1}, y_i]$, f is convex when i is even, and is concave when i is odd. We also use the notation $y_0 := 1$ and $y_{s+1} := -1$. Denote

$$\Pi(x) := \prod_{i=1}^{s}(x - y_i). \tag{1.1}$$