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B-spline approximation of elliptic problems with non-smooth coefficients

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Abstract

Weighted B-splines approximate smooth solutions of elliptic problems with maximal order. However, lack of regularity due to non-smooth coefficients of the partial differential equation can cause a severe loss of accuracy. A typical model problem is

$$-\nabla \cdot (\alpha \nabla u) = f \quad \text{in} \quad \Omega, \quad u = 0 \quad \text{on} \quad \partial\Omega,$$

where α is discontinuous across a submanifold $\Gamma \subset \Omega$. We show for the two-dimensional case, that the optimal convergence rates can be retained if we augment addition so-called singular splines to the weighted spline basis. The singular splines are constructed with an implicit representation of Γ and can model the discontinuous gradients of solutions accurately. As a result we obtain a meshless method of optimal order with the computational advantages of the B-spline calculus.

Keywords: B-spline approximation, meshless regular grid, finite elements, interface problems.

MSC: Primary 41A15; Secondary 65M60, 65L60.