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# Duality results for a general trigonometric approximation problem

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## Abstract

Let  $\alpha \in (1, \infty)$  and  $\mu$  be a regular finite Borel measure on a locally compact abelian group. The paper deals with a general trigonometric approximation problem in  $L^\alpha(\mu)$ , which arises in prediction theory of harmonizable symmetric  $\alpha$ -stable processes. To solve it, a duality method is applied, which is due to Nakazi and was generalized by Miamee and Pourahmadi and in the sequel successfully applied by several authors. The novelty of the present paper is that we do not make any additional assumption on  $\mu$ . Moreover, for  $\alpha = 2$ , multivariate extensions are obtained.

**Keywords:** regular Borel measure, space of  $\alpha$ -integrable functions, trigonometric approximation, duality.

**MSC:** Primary 42A10; Secondary 60G25, 43A15, 41A65.

## §1. Introduction

Let  $w$  be a weight function on  $(-\pi, \pi]$ ,  $S$  a proper subset of the set  $\mathbb{Z}$  of integers,  $x \in \mathbb{Z} \setminus S$ , and  $\mathcal{T}(S)$  the linear space of trigonometric  $S$ -polynomials, i. e. the linear space of all finite sums of the form  $\sum e^{is_j} u_j$ ,  $s_j \in S$ ,  $u_j \in \mathbb{C}$ . An important task in prediction theory of weakly stationary or, more generally, harmonizable symmetric  $\alpha$ -stable sequences is to compute the prediction error, i. e., the distance of the function  $e^{ix}$  to the set  $\mathcal{T}(S)$  with

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