



# Characterizations of almost greedy and partially greedy bases<sup>†</sup>

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## Abstract

We shall present new characterizations of partially greedy and almost greedy bases. A new class of basis (which we call reverse partially greedy basis) arises naturally from these characterizations of partially greedy bases.

**Keywords:** almost greedy basis, partially greedy basis, reverse partially greedy basis.

**MSC:** 46B15, 41A65.

## §1. Introduction

Let  $X$  be a real Banach space with a seminormalized (Schauder) basis  $(e_n)$  and biorthogonal functionals  $(e_n^*)$ .  $X$  is allowed to be finite-dimensional, in which case  $(e_n)$  is a finite algebraic basis for  $X$ . For any  $x \in X$ , set  $\text{supp}(x) = \{n : e_n^*(x) \neq 0\}$ . For a given finite set  $A \subset \mathbb{N}$ , let  $|A|$  denote the cardinality of  $A$ ,  $1_A = \sum_{i \in A} e_i$ , and  $P_A(x) = \sum_{i \in A} e_i^*(x)e_i$ . For finite sets  $A, B \subset \mathbb{N}$  we write  $A < B$  if  $\max A < \min B$ .

In [8] Konyagin and Temlyakov introduced the **thresholding greedy algorithm** (TGA)  $(G_m)$ , where  $G_m(x)$  is obtained by taking  $m$  of the largest coefficients in the

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