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A Bernstein inequality for differential and integral operators on Orlicz spaces[†]

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Abstract

In this paper we obtain a Bernstein inequality for polynomial differential operators and polynomial integral operators on Orlicz spaces. Let $\Phi : [0, +\infty) \to [0, +\infty]$ be an arbitrary Young function, K be an arbitrary compact set in $\mathbb{R}$ and $P(x)$ be a polynomial. Then there exists a constant C independent of Φ such that

$$\|P^m(D)f\|_{(\Phi)} \leq Cm \sup_{x\in K} |P^m(x)| \|f\|_{(\Phi)}$$

for all $m \in \mathbb{N}$ and all $f \in \mathcal{L}_{\Phi,K}$, where $\mathcal{L}_{\Phi,K} = \{f \in L^{\Phi}(\mathbb{R}) : \text{ supp}\widehat{f} \subset K\}$, $\widehat{f}$ is the Fourier transform of f and $\|\cdot\|_{(\Phi)}$ is the Luxemburg norm.

The corresponding result for polynomial integral operators and an application are also given.

Keywords: Orlicz spaces, generalized functions, Bernstein inequality.

MSC: 26D10, 46E30.

§1. Introduction

Let $1 \leq p \leq \infty$, $r > 0$, $f \in L^p(\mathbb{R})$ and $\text{sp}(f) \subset [-r, r]$. Then we have the following Bernstein inequality [7, 16]:

$$\|Df\|_p \leq r\|f\|_p, \tag{1.1}$$

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