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Reconstructing piecewise smooth bivariate functions from scattered data

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Abstract

Given scattered data values of a piecewise smooth function f within a domain Ω, we look for a piecewise adaptive approximation to f. Approximation techniques for scattered data approximation, as Radial Basis Function (RBF) or Moving Least-Squares (MLS), achieve reduced approximation orders near the boundary of the domain and near the unknown curves of jump singularities of the function or its derivatives. The idea used here is that the approximation errors near the boundaries, and near a singularity curve, fully characterize the behavior of the function at these locations. We refer to these approximation error values as the signature of f. In this paper, we aim at using these values in order to define the approximation. Assuming smoothness of the singularity curve, we suggest using a signed-distance approach to construct an approximation of the singularity curve. Now we find approximations to the different smooth segments of f, based upon matching the signatures of the approximant to the signature of f. The resulting approximation captures the singularity of the given data. As a result, the error in this first stage approximation is smooth. Hence, a second stage corrected improved approximation is constructed using a global approximation to the error obtained in the first stage approximation.

Keywords: bivariate approximation, piecewise smooth functions, moving least squares.

MSC: Primary 41A05; Secondary 41A63.