Ejercicio 4 – Extraordinaria 2 – Curso 20/21

Estudiar si la matriz
$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & 0 & -3 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

es diagonalizable por semejanza. En su caso, obtener la matriz diagonal D y al menos la base de uno de los subespacios propios.

$$p(\lambda) = |A - \lambda I| = \begin{vmatrix} 1-\lambda & -1 & 0 & 0 \\ 0 & 1-\lambda & 0 & 3 \\ 0 & 2 & -\lambda & -3 \\ 0 & 1 & 0 & 1-\lambda \end{vmatrix} =$$

$$= (-1)^{3+3} \cdot (-\lambda) \cdot \begin{vmatrix} 1-\lambda & -1 & 0 \\ 0 & 1-\lambda & 3 \\ 0 & 1 & 1-\lambda \end{vmatrix} = (-\lambda)(1-\lambda) \begin{vmatrix} 1-\lambda & 3 \\ 1 & 1-\lambda \end{vmatrix} =$$

$$= (-\lambda)(1-\lambda)\left((1-\lambda)^2 - 3\right) = (-\lambda)(1-\lambda)(\lambda^2 - 2\lambda - 2) = 0$$

$$\lambda = 0 \qquad \lambda = 1 \qquad \lambda^2 - 2\lambda - 2 = 0$$

$$\lambda = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} =$$

$$= \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

Valores propios: $\left.\begin{array}{l} \lambda = 0 \\ \lambda = 1 \\ \lambda = 1 + \sqrt{3} \\ \lambda = 1 - \sqrt{3} \end{array}\right\}$ A es diagonalizable

$$V_0 = \{ x \mid (A - 0.I)x = 0 \}$$

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & 0 & -3 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{array}{l} x - y = 0 \\ y + 3t = 0 \\ 2y - 3t = 0 \\ y + t = 0 \end{array}\right\} \quad \begin{array}{l} \longrightarrow x = y = 0 \\ \longrightarrow 3y = 0 \Rightarrow y = 0 \\ \longrightarrow t = -y = 0 \\ z = \lambda, \ \forall \lambda \in \mathbb{R} \end{array}$$

$$B_{V_0} = \{ (0, 0, 1, 0) \}$$

$\underline{\lambda = 1}$

$$V_1 = \{ x \mid (A - 1.I)x = 0 \}$$

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 2 & -1 & -3 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{array}{r} -y = 0 \\ 3t = 0 \\ 2y - z - 3t = 0 \\ y = 0 \end{array}\right\} \quad \begin{array}{l} \longrightarrow y = 0 \\ \longrightarrow t = 0 \\ \longrightarrow z = 2y - 3t = 0 \\ x = \lambda, \ \lambda \in \mathbb{R} \end{array}$$

$$B_{V_1} = \{ (1, 0, 0, 0) \}$$

$$V_{1+\sqrt{3}} = \{ x \,/\, (A-(1+\sqrt{3})I)x = 0 \}$$

$$\begin{pmatrix} -\sqrt{3} & -1 & 0 & 0 \\ 0 & -\sqrt{3} & 0 & 3 \\ 0 & 2 & -1-\sqrt{3} & -3 \\ 0 & 1 & 0 & -\sqrt{3} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{array}{l} -\sqrt{3}x - y = 0 \\ -\sqrt{3}y + 3t = 0 \\ 2y + (-1-\sqrt{3})z - 3t = 0 \\ y - \sqrt{3}t = 0 \end{array}\right\}$$

$\rightarrow -\sqrt{3}x - \sqrt{3}t = 0$

$\Rightarrow x + t = 0 \Rightarrow x = -t$

$\rightarrow (-\sqrt{3})(\sqrt{3})t + 3t = 0$

$\quad = -3t + 3t = 0$

$\rightarrow y = \sqrt{3}t$

$\rightarrow (-1-\sqrt{3})z = 3t - 2y = 3t - 2\sqrt{3}t \Rightarrow$

$$\Rightarrow z = \frac{3-2\sqrt{3}}{-1-\sqrt{3}}t = \frac{(3-2\sqrt{3})(-1+\sqrt{3})}{(-1-\sqrt{3})(-1+\sqrt{3})}t =$$

$$= \frac{-3+3\sqrt{3}+2\sqrt{3}-6}{-2}t =$$

$$= \left(\frac{9}{2} - \frac{5\sqrt{3}}{2}\right)t$$

$t = \lambda$

$$\left.\begin{array}{l} x = -\lambda \\ y = \sqrt{3}\lambda \\ z = \left(\frac{9}{2} - \frac{5\sqrt{3}}{2}\right)\lambda \\ t = \lambda \end{array}\right\}$$

$$B_{V_{1+\sqrt{3}}} = \left\{ \left(-1, \sqrt{3}, \left(\frac{9}{2} - \frac{5\sqrt{3}}{2}\right), 1\right) \right\}$$