Ejercicio 5 – Convocatoria Ordinaria 1 – Curso 2020/21

Calcular la solución mínimo-cuadrática de norma mínima del sistema AX = b donde,

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \\ -2 & 4 & -2 \end{pmatrix} \quad \text{y} \quad b = \begin{pmatrix} 0 \\ -2 \\ 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} x - 2y \qquad &= 0 \\ -3z &= -2 \\ -2x + 4y - 2z &= 0 \end{aligned}\right\}$$

$$A' = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & -3 \\ -2 & 4 & -2 \end{pmatrix}$$

$$|A'| = 0 - 12 + 0 - 0 - 0 + 12 = 0$$

$$\Downarrow$$

$$\begin{vmatrix} -2 & 0 \\ 0 & -3 \end{vmatrix} = 6 \neq 0$$

$$\Downarrow$$

$$rg(A') = 2$$

$$(A'|b') = \begin{pmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & -3 & -2 \\ -2 & 4 & -2 & 0 \end{pmatrix}$$

$$\begin{vmatrix} -2 & 0 & 0 \\ 0 & -3 & -2 \\ 4 & -2 & 0 \end{vmatrix} = 8 \neq 0$$

$$\Downarrow$$

$$rg(A'|b') = 3$$

Como $rg(A') = 2 \neq 3 = rg(A'|b') \Rightarrow$ S. Incompatible

$$(A|I) = \begin{pmatrix} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 0 & 1 & 0 \\ -2 & 4 & -2 & 0 & 0 & 1 \end{pmatrix} \sim_f \begin{pmatrix} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 0 & 1 & 0 \\ 0 & 0 & -2 & 2 & 0 & 1 \end{pmatrix} \sim_f$$

$F_3 \leftrightarrow F_3 + 2F_1$

$F_2 \leftrightarrow \frac{-1}{3} F_2$

$$\sim_f \begin{pmatrix} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & -2 & 2 & 0 & 1 \end{pmatrix} \sim_f \left(\underbrace{\begin{matrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix}}_{=H} \middle| \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & -1/3 & 0 \\ 2 & -2/3 & 1 \end{matrix}}_{=Q} \right)$$

$$F_3 \to F_3 + 2F_2$$

$$|Q| = -\frac{1}{3}$$

$$F = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Adj(Q) = \begin{pmatrix} -1/3 & 0 & 2/3 \\ 0 & 1 & 2/3 \\ 0 & -1 & -1/3 \end{pmatrix} \Rightarrow Adj(Q)^t = \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1 & -1 \\ 2/3 & 2/3 & -1/3 \end{pmatrix}$$

$$Q^{-1} = \frac{Adj(Q)^t}{|Q|} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 3 \\ -2 & -2 & 1 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 0 \\ 0 & -3 \\ -2 & -2 \end{pmatrix} \qquad A = E.F$$

$$A^+ = F^R . E^L$$

$$F^R = F^t \left(FF^t\right)^{-1}$$

$$FF^t = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix} \qquad |FF^t| = 5$$

$$Adj\left(FF^t\right)^t = \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix} \Rightarrow \left(FF^t\right)^{-1} = \begin{pmatrix} 1/5 & 0 \\ 0 & 1 \end{pmatrix}$$

$$F^R = \begin{pmatrix} 1 & 0 \\ -2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/5 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 1 & 0 \\ -2 & 0 \\ 0 & 5 \end{pmatrix}$$

$$E^L = (E^tE)^{-1} \cdot E^t$$

$$E^tE = \begin{pmatrix} 1 & 0 & -2 \\ 0 & -3 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -3 \\ -2 & -2 \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 4 & 13 \end{pmatrix} \Rightarrow |E^tE| = 49$$

$$\mathrm{Adj}(E^tE) = \begin{pmatrix} 13 & -4 \\ -4 & 5 \end{pmatrix} \Rightarrow \mathrm{Adj}(E^tE)^t = \mathrm{Adj}(E^tE)$$

$$E^tE^{-1} = \frac{1}{49} \begin{pmatrix} 13 & -4 \\ -4 & 5 \end{pmatrix}$$

$$E^L = \frac{1}{49} \begin{pmatrix} 13 & -4 \\ -4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 & -2 \\ 0 & -3 & -2 \end{pmatrix} = \frac{1}{49} \begin{pmatrix} 13 & 12 & -18 \\ -4 & -15 & -2 \end{pmatrix}$$

$$A^+ = F^R E^L = \frac{1}{5} \begin{pmatrix} 1 & 0 \\ -2 & 0 \\ 0 & 5 \end{pmatrix} \cdot \frac{1}{49} \begin{pmatrix} 13 & 12 & -18 \\ -4 & -15 & -2 \end{pmatrix} =$$

$$= \frac{1}{245} \begin{pmatrix} 13 & 12 & -18 \\ -26 & -24 & 36 \\ -20 & -75 & -10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^+ \cdot B = \frac{1}{245} \begin{pmatrix} 13 & 12 & -18 \\ -26 & -24 & 36 \\ -20 & -75 & -10 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} = \frac{1}{245} \begin{pmatrix} -24 \\ 48 \\ 150 \end{pmatrix}$$

$$x = -\frac{24}{245} \qquad y = \frac{48}{245} \qquad z = \frac{150}{245}$$