Ejercicio 3 – Extraordinaria 2 – Curso 20/21

a) Definir el álgebra de Boole $\mathbb{B}_2^n$ y el orden inducido en ella.

b) Definir y dibujar un circuito lógico que calcule (x–4) módulo 4, siendo x un entero comprendido entre 0 y 7.

b) $f : \mathbb{B}_2^n \longrightarrow \mathbb{B}_2^m$

$0 = (0\,0\,0)_2$

$1 = (0\,0\,1)_2$

$2 = (0\,1\,0)_2$

$3 = (0\,1\,1)_2$

$4 = (1\,0\,0)_2$

$5 = (1\,0\,1)_2$

$6 = (1\,1\,0)_2$

$7 = (1\,1\,1)_2$

$f(x) = (x-4) \bmod 4$

$\mathbb{Z}_4 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}\}$

$0 = (0\,0)_2$

$1 = (0\,1)_2$

$2 = (1\,0)_2$

$3 = (1\,1)_2$

$f : \mathbb{B}_2^3 \longrightarrow \mathbb{B}_2^2$

$f(0) = (0-4) \bmod 4 \equiv (-4) \bmod 4 \equiv 0 \bmod 4 = 0$

$-4 = 4 \cdot (-1) + 0$

$f(0,0,0) = (0,0)$

$f(1) = (1-4) \bmod 4 \equiv (-3) \bmod 4 \equiv 1 \bmod 4 = 1$

$-3 = 4 \cdot (-1) + \underline{\underline{1}}$

$f(0,0,1) = (0,1)$

$f(2) = (2-4) \bmod 4 \equiv (-2) \bmod 4 \equiv 2 \bmod 4 = 2$

$-2 = 4 \cdot (-1) + 2$

$f(0,1,0) = (1,0)$

$$f(3) = (3-4) \bmod 4 \equiv -1 \bmod 4 \equiv 3 \bmod 4 = 3$$

$$f(0,1,1) = (1,1)$$

$$f(4) = (4-4) \bmod 4 \equiv 0 \bmod 4$$

$$f(1,0,0) = (0,0)$$

$$f(5) = (5-4) \bmod 4 \equiv 1 \bmod 4$$

$$f(1,0,1) = (0,1)$$

$$f(6) = (6-4) \bmod 4 \equiv 2 \bmod 4$$

$$f(1,1,0) = (1,0)$$

$$f(7) = (7-4) \bmod 4 \equiv 3 \bmod 4$$

$$f(1,1,1) = (1,1)$$

x	y	z	f_1	f_2
0	0	0	0	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

$$f_1(x,y,z) = y$$

$$f_2(x,y,z) = z$$

$$f(x,y,z) = (y,z)$$