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Polynomial operators and local smoothness classes on the unit interval, II[†]

H. N. Mhaskar

Abstract

We prove the existence of quadrature formulas exact for integrating high degree polynomials with respect to Jacobi weights based on scattered data on the unit interval. We also obtain a characterization of local Besov spaces using the coefficients of a tight frame expansion.

Keywords: Local approximation, localized Jacobi kernels, quadrature formulas, tight frames, local Besov spaces.

MSC: Primary 41A55; Secondary 41A25, 42B35.

§1. Introduction

It is well known that a major drawback of polynomial approximation is that polynomials cannot be localized; a polynomial of degree n is completely determined by its values at $n + 1$ points on an interval, howsoever small. Another example is the following. If P_n^* is a best polynomial approximation of degree at most n to the function $f(x) = |x|$ on $[-1, 1]$, then there are at least $n + 2$ points y_j on $[-1, 1]$, where $|f(y_j) - P_n^*(y_j)| \geq cn^{-1}$, where c is a positive constant independent of n and the points y_j . Moreover, as $n \rightarrow \infty$, these points become dense on $[-1, 1]$. Thus, even though the function is piecewise analytic, the

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F. Marcellán

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