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# On the difficulty of preserving monotonicity via projections and related results

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**Abstract**

A subspace $V$ of a Banach space $X$ is said to be *complemented* if there exists a (bounded) projection mapping $X$ onto $V$. Obviously all subspaces of finite-dimension are complemented. The goal of this note is to show that there are (relatively) few *monotonically complemented* subspaces of finite-dimension in $X = (C[a,b], \|\cdot\|_\infty)$; that is, finite-dimensional subspaces $V \subset X$ for which there exists a projection $P : X \to V$ such that $Pf$ is monotone-increasing whenever $f$ is. We obtain several corollaries from this consideration, including a result describing the difficulty of preserving $n$-*convexity* via a projection.

**Keywords:** Projections, monotonicity, shape-preservation.

**MSC:** Primary 41A29; Secondary 41A65.

## §1. Introduction and preliminaries

By a *cone* $S$ of a (real) Banach space $X$ we mean a convex subset of $X$ which is closed under nonnegative scalar multiplication. Every cone $S \subset X$ contains the origin and a *pointed* cone contains no lines through the origin. Let $\mathcal{L}(X)$ denote the set of linear operators on $X$. For a given cone $S$ and an operator $Q \in \mathcal{L}(X)$, a natural question arises: does $Q$ leave $S$ invariant? There are numerous settings in which knowing the answer to this has