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# Bipolar varieties and real solving of a singular polynomial equation<sup>†</sup>

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## Abstract

We introduce the concept of a *bipolar* variety of a real algebraic hypersurface. This notion is then used for the design and complexity estimations of a novel type of algorithms that finds algebraic sample points for the connected components of a singular real hypersurface. The complexity of these algorithms is polynomial in the maximal geometric degree of the bipolar varieties of the given hypersurface and in this sense intrinsic.

**Keywords:** Real polynomial equation solving; singular hypersurface; polar varieties.

**MSC:** Primary 14P05, 14Q10, 14B05; Secondary 14Q15, 68W30.

## §1. Introduction

This paper is based on the concept of polar varieties which classically goes back to F. Severi and J. A. Todd in the 1930's and beyond that to the work of J.-V. Poncelet in the period of 1813–1829. The modern theory started in 1975 with essential contributions

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