ISSN: 1889-3066
Jaen J. Approx. 3(1) (2011), 1–14
Web site: jja.ujaen.es
Jaen Journal on Approximation

Some inequalities related to the moduli of smoothness of polynomials

Jorge Bustamante

Communicated by
M. A. Jiménez

Received
May 11, 2010
Accepted
February 22, 2011

Abstract

We present some inequalities related to the moduli of smoothness of trigonometric and algebraic polynomials. In the case of algebraic polynomials we consider the Ditzian-Totik type moduli of smoothness. Some inequalities are similar to the ones due to Nikolskii and Stechkin and they are also a connected with Bernstein type inequalities.

Keywords: Polynomial inequalities, modulus of smoothness, Bernstein's inequality, Nikolskii-Stechkin's inequality.

MSC: Primary 41A17; Secondary 26D05.

§1. Introduction

Let $C[-1,1]$ be the space of all real continuous functions f endowed with the norm $\|f\|_\infty = \sup_{x\in[-1,1]}|f(x)|$. For $1 \le p < \infty$, let $L_p[-1,1]$ be the usual space of Lebesgue integrable functions with the norm $\|f\|_p = \left(\int_{-1}^1 |f(x)|^p\, dx\right)^{1/p}$. To simplify notations we sometimes write $L_\infty[-1,1]$ instead of $C[-1,1]$. The family of all algebraic polynomials of degree not greater than n is denoted by Π_n.

The function $\varphi(x) = \sqrt{1-x^2}$, $x \in [-1,1]$, will be used throughout the paper. For $r \in \mathbb{N}$ and $f \in L_p[-1,1]$ $(1 \le p \le \infty)$ the Ditzian-Totik modulus of smoothness of order r