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A contribution to the Grünwald–Marcinkiewicz theorem†

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Abstract

This paper is a certain generalization of the Grünwald–Marcinkiewicz theorem revealing its connection to a process defined by S. N. Bernstein.

Keywords: interpolation, operator norm, divergence, Grünwald-Marcinkiewicz theorem, Bernstein operator.

MSC: 41A05.

§1. Introduction

1.1. We begin with some definitions and notations. $\tilde{C}$ stands for the space of 2π-periodic continuous functions, $\mathcal{T}_m$ denotes the space of trigonometric polynomials of degree at most m of the form $\frac{a_0}{2}+\sum_{k=1}^{m}(a_k\cos k\vartheta+b_k\sin k\vartheta)$, a_k, b_k being reals. If $\Theta=\{\vartheta_{km}, k=0,\dots,2m, m=1,2,\dots\}\subset[0,2\pi)$ is an interpolatory matrix with

$$0\le\vartheta_{0m}<\vartheta_{1m}<\cdots<\vartheta_{2m,m}<2\pi,$$

then the uniquely defined m^{th} trigonometric interpolatory polynomial for $f\in\tilde{C}$ is

$$T_m(f,\Theta,\vartheta)=\sum_{k=0}^{2m}f(\vartheta_{km})t_{km}(\Theta,\vartheta),$$

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