ISSN: 1889-3066

Web site: jja.ujaen.es

Jaen J. Approx. 5(1) (2013), 81–99

Jaen Journal on Approximation

Best local approximation and lateral differentiability[†]

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Communicated by
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Received November 9, 2012
Accepted September 24, 2013

Abstract

In this paper we give sufficient conditions over the differentiability of a function to assure existence of the best local approximant in L^p-spaces, $0 < p \leq \infty$. These conditions are weaker than those given in previous works. For $p = 2$ we show that, in certain way, they are also necessary. In addition, we characterize the best local approximant.

Keywords: best approximation, L^p-norm, local approximant, lateral differentiability.

MSC: Primary 41A50; Secondary 41A10.

§1. Introduction.

Let $x_1, x_2, ..., x_k$ be k points in $\mathbb{R}$ and let $a > 0$ be such that the intervals

$$A_{a,i} := [x_i - a, x_i + a],\ 1 \leq i \leq k,$$

are pairwise disjoint. Let $\mathcal{L}$ be the space of equivalence class of Lebesgue measurable real functions defined on $A_a := \bigcup_{i=1}^{k} A_{a,i}$. For each Lebesgue measurable set $A \subset A_a$, with

[†]This work was supported by Universidad Nacional de Rio Cuarto and Conicet.