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Whitney's theorem for local anisotropic polynomial L_p-approximation, $0 < p < 1$[†]

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Abstract

Dinh Dũng and T. Ullrich have proven a multivariate Whitney's theorem for the local anisotropic polynomial approximation in $L_p(Q)$ for $1 \le p \le \infty$, where Q is a d-parallelepiped in $\mathbb{R}^d$ with sides parallel to the coordinate axes. They considered the error of best approximation of a function f by algebraic polynomials of fixed degree at most $r_i - 1$ in the variable x_i, $i = 1, ..., d$. The convergence rate of the approximation error when the size of Q goes to $\mathbf{0}$ is characterized by a so-called total mixed modulus of smoothness. The method of proof used by these authors is not suitable for the case $0 < p < 1$. In the present paper, by a different technique we prove this theorem for $0 < p \le \infty$.

Keywords: Whitney's theorem, anisotropic approximation by polynomials, total mixed modulus of smoothness, Marchaud's inequality.

MSC: 41A10, 41A50, 41A63.

§1. Introduction

Let $\omega_r(f, \cdot)_{p,I}$ be the rth modulus of smoothness of a function $f \in L_p(I)$ and let $E_r(f)_{p,I}$ be the error of best L_p-approximation of f by algebraic polynomials of degree at most

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