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On Shapiro's lethargy theorem and some applications

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Abstract

Shapiro's lethargy theorem [48] states that if $\{A_n\}$ is any non-trivial linear approximation scheme on a Banach space X, then the sequences of errors of best approximation $E(x, A_n) = \inf_{a\in A_n} \|x - a_n\|_X$ may decay almost arbitrarily slowly. Recently, Almira and Oikhberg [11, 12] investigated this kind of result for general approximation schemes in the quasi-Banach setting. In this paper, we consider the same question for F-spaces with non decreasing metric d. We also provide applications to the rate of decay of s-numbers, entropy numbers and slow convergence of sequences of operators.

Keywords: Approximation scheme, approximation error, approximation with restrictions, Bernstein's Lethargy Theorem, Shapiro's Theorem, Metric vector space, F-space.

MSC: Primary 41A29; Secondary 41A25, 41A65, 41A27.

§1. Motivation

A famous theorem by Kakutani [30] states that a topological vector space is metrizable if and only if it contains a countable basis of neighborhoods. Furthermore, if the topological vector space X admits a compatible metric d then it also admits an equivalent metric d^* which is translation invariant. Thus, we assume in all what follows that our metrics