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Polynomial spaces revisited via weight functions

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Abstract

Extended Chebyshev spaces are natural generalisations of polynomial spaces due to the same upper bounds on the number of zeros. In a natural approach, many results of the polynomial framework have been generalised to the larger Chebyshevian framework, concerning Approximation Theory as well as Geometric Design. In the present work, we go the reverse way: considering polynomial spaces as examples of Extended Chebyshev spaces, we apply to them results specifically developed in the Chebyshevian framework. On a closed bounded interval, each Extended Chebyshev space can be defined by means of sequences of generalised derivatives which play the same role as the ordinary derivatives for polynomials. We recently achieved an exhaustive description of the infinitely many such sequences. Surprisingly, this issue is closely related to the question of building positive linear operators of the Bernstein type. As Extended Chebyshev spaces, one can thus search for all generalised derivatives which can be associated with polynomial spaces on closed bounded intervals. Though this may a priori seem somewhat nonsensical on account of the simplicity of the ordinary derivatives, this actually leads to new interesting results on polynomial and rational Bernstein operators and related results of convergence.

Keywords: Bernstein bases, Bernstein operators, Extended Chebyshev spaces, generalised derivatives, weight functions, blossoms, polynomial and rational spaces.

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