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Finite dimensional Chebyshev subspaces of spaces of discontinuous functions

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Abstract

In this paper the author studies the existence and the characterization of the n dimensional Chebyshev subspaces of $L_\infty[a, b]$, $B[a, b]$ and some other spaces of discontinuous functions. In the case when the space admits an n dimensional Chebyshev subspace, the author develops a complete characterization for those n dimensional Chebyshev subspaces. In the case when the space does not admit an n dimensional Chebyshev subspaces, the author proves it.

Keywords: Chebyshev spaces, best approximation, Banach lattice, spaces of discontinuous functions.

MSC: 41A65.

§1. Introduction

If A is a subset of the normed linear space X, then the distance between $x \in X$ and A, $\mathrm{d}(x, A)$, is defined to be $\min\{\|x - y\| : y \in A\}$. A is said to be proximinal in X if for each $x \in X$ there is a point $y_0 \in A$ such that $\|x - x_0\| = \mathrm{d}(x, A)$; the element y_0 is called a best approximation for x from A. If for each $x \in X$, the best approximation for x from A is unique, then the subset A is called a Chebyshev subset of X. If Q is a compact subset of the real numbers, then $C(Q)$ denotes the Banach space of all continuous real valued functions defined on Q equipped with the uniform norm, that is $\|f\| = \max\{|f(x)| : x \in Q\}$.