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Simultaneous Gaussian quadrature for Angelesco systems[†]

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Abstract

We investigate simultaneous Gaussian quadrature for two integrals of the same function f but on two disjoint intervals. The quadrature nodes are zeros of a type II multiple orthogonal polynomial for an Angelesco system. We recall some known results for the quadrature nodes and the quadrature weights and prove some new results about the convergence of the quadrature formulas. Furthermore we give some estimates of the quadrature weights. Our results are based on a vector equilibrium problem in potential theory and weighted polynomial approximation.

Keywords: Simultaneous quadrature, Angelesco systems, Hermite-Padé approximation, vector potential equilibrium, weighted polynomial approximation..

MSC: Primary 41A55; Secondary 41A28, 31A15, 65D32.

§1. Introduction

1.1. Simultaneous Gaussian quadrature

Suppose we are given r measures μ_1, \dots, μ_r on the real line and a function $f : \mathbb{R} \rightarrow \mathbb{R}$ and that we want to approximate the integrals

$$\int f(x) d\mu_j(x), \quad 1 \leq j \leq r,$$

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