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Summability of Fourier transforms in variable Hardy and Hardy-Lorentz spaces†

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Abstract

Let $p(\cdot) : \mathbb{R}^n \to (0,\infty)$ be a variable exponent function satisfying the globally log-Hölder condition and $0 < q \leq \infty$. We introduce the variable Hardy and Hardy-Lorentz spaces $H_{p(\cdot)}(\mathbb{R}^d)$ and $H_{p(\cdot),q}(\mathbb{R}^d)$. A general summability method, the so called θ-summability is considered for multi-dimensional Fourier transforms. Under some conditions on θ, it is proved that the maximal operator of the θ-means is bounded from $H_{p(\cdot)}(\mathbb{R}^d)$ to $L_{p(\cdot)}(\mathbb{R}^d)$ and from $H_{p(\cdot),q}(\mathbb{R}^d)$ to $L_{p(\cdot),q}(\mathbb{R}^d)$. This implies some norm and almost everywhere convergence results for the θ-means, amongst others the generalization of the well known Lebesgue's theorem. Some special cases of the θ-summation are considered, such as the Riesz, Bochner-Riesz, Weierstrass, Picard and Bessel summations.

Keywords: variable Hardy spaces, variable Hardy-Lorentz spaces, atomic decomposition, θ-summability, maximal operator.

MSC: Primary 42B08; Secondary 42A38, 42A24, 42B25, 42B30.

§1. Introduction

It was proved by Lebesgue [23] that the Fejér means [10] of the trigonometric Fourier transforms of a one-dimensional integrable function $f \in L_1(\mathbb{R})$ converge almost every-

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