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On the degree of rational approximation of Markov functions on discrete sets

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Dedicated to the memory of my dear teacher, Professor Valery V. Vavilov.

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Abstract

This article is devoted to results of rational approximation of the Markov function

$$\hat{\alpha}(z) = \int_F \frac{d\alpha(x)}{z-x},$$

where α is a positive Borel measure with support $\operatorname{supp}\alpha = F = [a,b] \subset (0,\infty)$ and $d\alpha/dx > 0$ a.e. on F (with respect to the Lebesgue measure). We study asymptotic properties of the best uniform rational approximation of Markov functions $\hat{\alpha}$ on point systems $E_N \subset (-\infty, 0)$ when the number of points N in the set E_N and the degree of rational approximants n satisfy an asymptotic relation $N/n \to \theta > 2$ as $n \to \infty$. The degree of rational approximation is described in terms of the solutions of certain logarithmic potential-theoretic problems, central among which is a minimal energy problem in the presence of an external field. We also investigate the limit distribution of poles of the best rational approximants and of points of Chebyshev alternance.

Keywords: rational approximation, Markov function, degree of approximation, approximation on discrete sets, Hankel operator, potential theory.

MSC: Primary 30E10; Secondary 41A20, 41A25.