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Nonlinear difference equations for a modified Laguerre weight: Laguerre-Freud equations and asymptotics

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Abstract

In this paper we derive second and third order nonlinear difference equations for one of the recurrence coefficients in the three term recurrence relation of polynomials orthogonal with respect to a modified Laguerre weight. We show how these equations can be obtained from the Bäcklund transformations of the third Painlevé equation. We also show how to use nonlinear difference equations to derive a few terms in the formal asymptotic expansions in n of the recurrence coefficients.

Keywords: orthogonal polynomials, difference equations, Painlevé equations, Bäcklund transformations, asymptotic expansions.

MSC: 33C47, 39A99, 34E05, 42C05.

§1. Introduction

In recent years there has been a considerable interest to derive linear difference-differential equations for polynomials which are orthogonal with respect to a weight having a rational logarithmic derivative - the semi-classical weights, and nonlinear difference equations for their recurrence coefficients (e.g., [1, 2, 4, 3] and the references therein). In the present paper we will use two methods, commonly used in the literature of special functions:

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