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Application of Zernike polynomials in solving certain first and second order partial differential equations

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Abstract

Integration operational matrix methods based on Zernike polynomials are used to determine approximate solutions of a class of non-homogeneous partial differential equations (PDEs) of first and second order. Due to the nature of the Zernike polynomials being described in the unit disk, this method is particularly effective in solving PDEs over a circular region. Further, the proposed method can solve PDEs with discontinuous Dirichlet and Neumann boundary conditions, and as these discontinuous functions cannot be defined at some of the Chebyshev or Gauss-Lobatto points, the much acclaimed pseudo-spectral methods are not directly applicable to such problems. Solving such PDEs is also a new application of Zernike polynomials as so far the main application of these polynomials seem to have been in the study of optical aberrations of circularly symmetric optical systems. In the present method, the given PDE is converted to a system of linear equations of the form $A\mathbf{x} = \mathbf{b}$ which may be solved by both l_1 and l_2 minimization methods among which the l_1 method is found to be more accurate. Finally, in the expansion of a function in terms of Zernike polynomials, the rate of decay of the coefficients is given for certain classes of functions.

Keywords: integration operational matrix, Laplace equation, partial differential equations, Zernike polynomials.

MSC: Primary 35AXX; Secondary 41A10.