



A solution to Stechkin’s problem for functions of a self-adjoint operator in a Hilbert space, Taikov-type inequalities, and their applications[†]

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Abstract

In this paper we solve the problem of approximating functionals $(\varphi(A)x, f)$ (where $\varphi(A)$ is some function of self-adjoint operator A) on the class of elements of a Hilbert space that is defined using another function $\psi(A)$ of the operator A . In addition, we obtain a series of sharp Taikov-type additive inequalities that estimate $|(\varphi(A)x, f)|$ with the help of $\|\psi(A)x\|$ and $\|x\|$. We also present several applications of the obtained results. First, we find sharp constants in inequalities of the type used in Hörmander theorem on comparison of operators in the case when operators are acting in a Hilbert space and are functions of a self-adjoint operator. Second, we obtain Taikov-type inequalities for functions of the operator $\frac{1}{i} \frac{d}{dt}$ in the spaces $L_2(\mathbb{R})$ and $L_2(\mathbb{T})$, as well as for integrals with respect to spectral measures, defined with the help of classical orthogonal polynomials.

Keywords: functions of operators, Landau-Kolmogorov inequalities, best approximation, orthogonal polynomials.

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