



Optimal bounds for the sine and hyperbolic tangent means III

Monika Nowicka and Alfred Witkowski

Abstract

We provide the optimal bounds for the sine and hyperbolic tangent means in terms of various weighted means of the arithmetic and maximum means.

Keywords: Seiffert-like mean, Seiffert function, convex function.

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S. Tikhonov

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§1. Introduction, definitions and notations

The means

$$M_{\sin}(x, y) = \begin{cases} \frac{x-y}{2 \sin \frac{x-y}{x+y}}, & x \neq y, \\ x, & x = y \end{cases} \quad (\text{sine mean})$$

and

$$M_{\tanh}(x, y) = \begin{cases} \frac{x-y}{2 \tanh \frac{x-y}{x+y}}, & x \neq y, \\ x, & x = y \end{cases} \quad (\text{hyperbolic tangent mean})$$