ISSN: 1889-3066

Web site: jja.ujaen.es

Jaen J. Approx. 13 (2022), 43–73

Jaen Journal on Approximation

Approximation of functions on manifolds in high dimension from noisy scattered data†

Shira Faigenbaum-Golovin and David Levin

Communicated by M. Buhmann

Received November 15, 2022
Accepted November 21, 2022

Abstract

In this paper, we consider the fundamental problem of approximation of functions on a low-dimensional manifold embedded in a high-dimensional space. Classical approximation methods, developed for the low-dimensional case, are challenged by the high-dimensional data, and the presence of noise. Here, we introduce a new approximation method that is parametrization free, can handle noise and outliers in both the scattered data and function values and does not require any assumptions on the scattered data geometry. Given a noisy point-cloud situated near a low dimensional manifold and the corresponding noisy function values, the proposed solution finds a noise-free, quasi-uniform manifold reconstruction as well as the denoised function values at these points. Next, this data is used to approximate the function at new points near the manifold. We prove that in the case of noise-free samples the approximation order is $O(h^2)$, where h depends on the local density of the dataset (i.e., the fill distance), and the function variation. We demonstrate the effectiveness of our approach by examining smooth and non-smooth functions on different manifold topologies, under various noise levels.

Keywords: approximation of functions, high dimensions, scattered data, noisy data, manifold learning, dimensional reduction.

MSC: Primary 41A63; Secondary 41-xx, 65D99.

†Shira is grateful to the Eric and Wendy Schmidt Fund for Strategic Innovation, the Zuckerman-CHE STEM Program, Duke University, and in part the Simons Foundation under Grant Math+X 400837 for supporting her research.